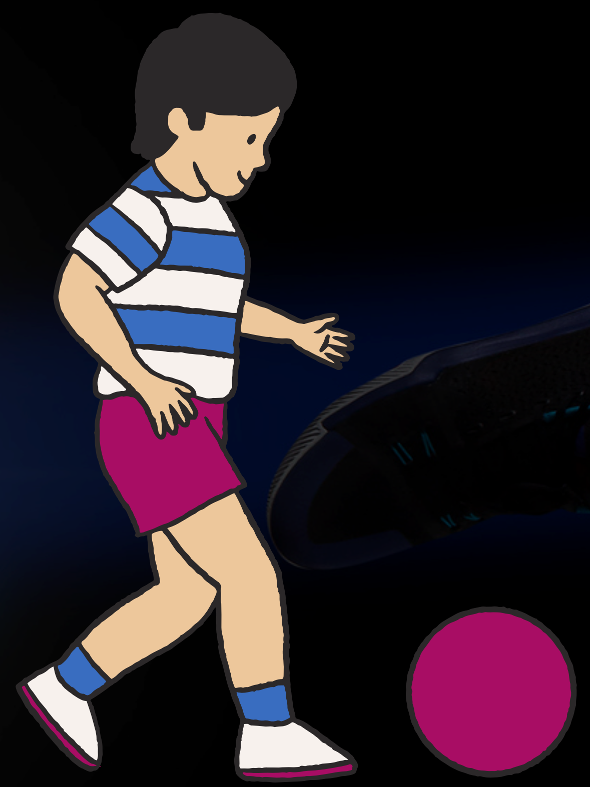
Prashant Kirad

MOTION IN A PLANE

Class-11th



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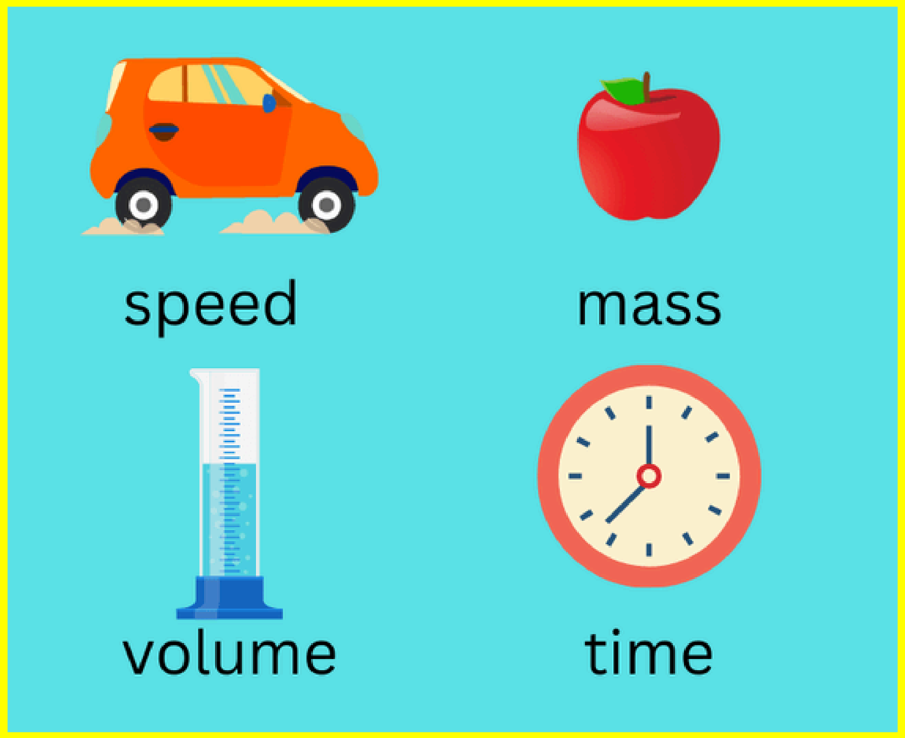
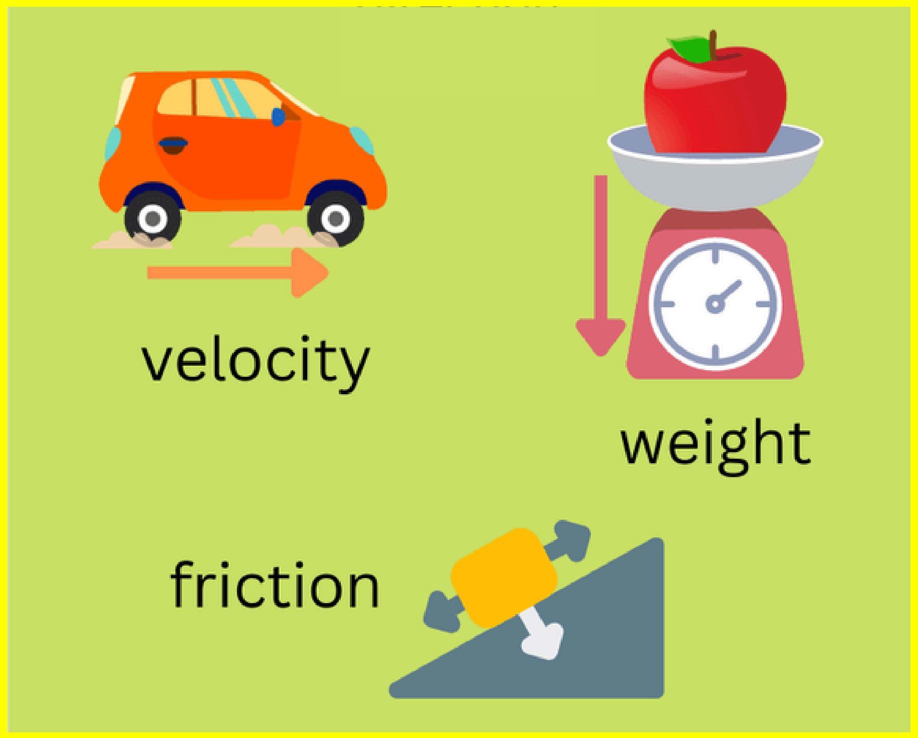
Physical Quantities

Scalar Quantity	Vector Quantity
A physical quantity that has only magnitude (value) but no direction. Do not follow the laws of vector algebra.	A physical quantity that has both magnitude and direction. Follows the laws of vector algebra.





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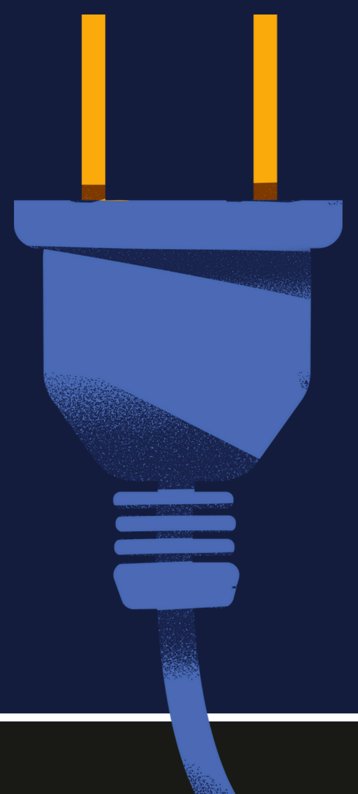
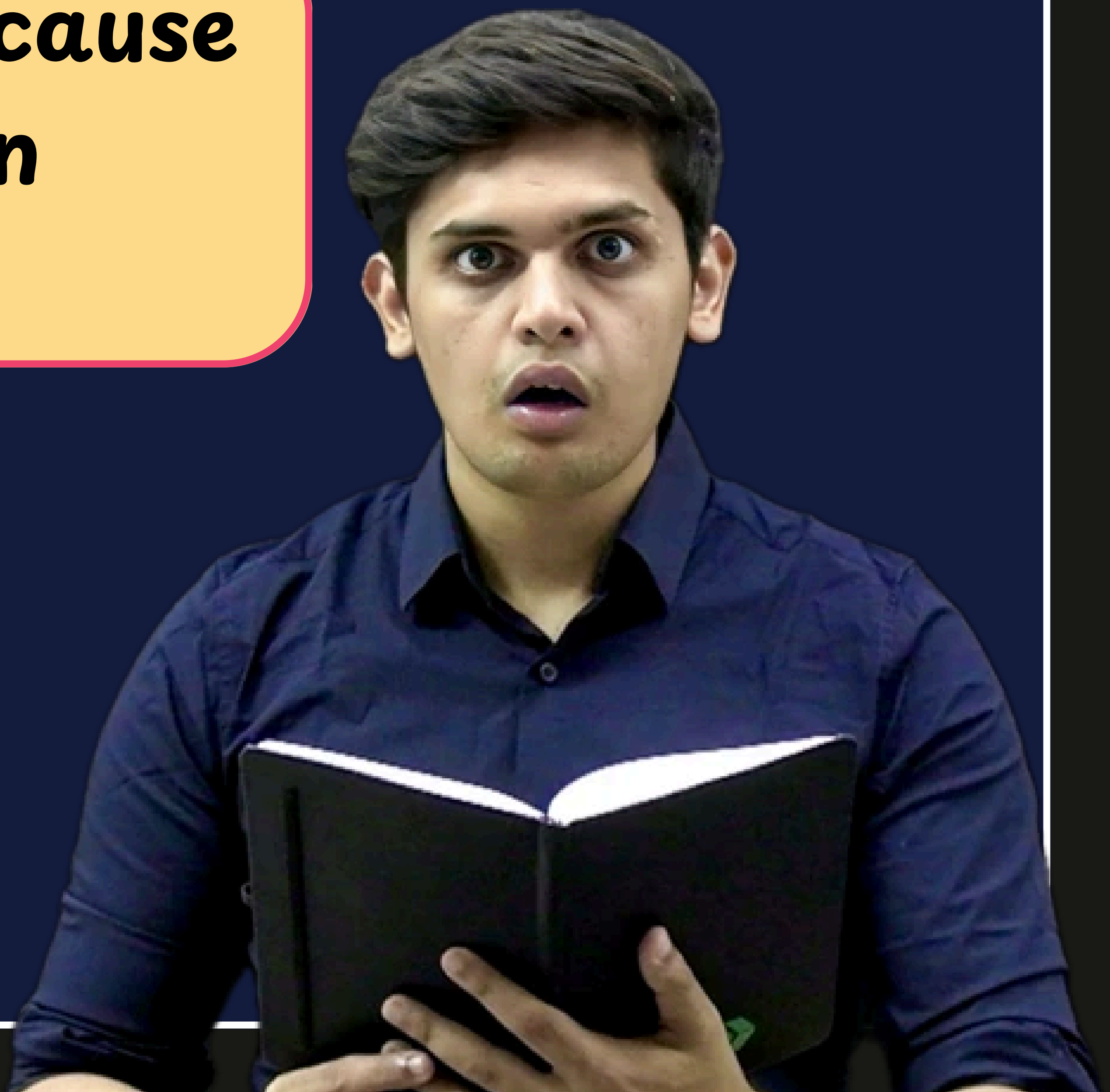
DID YOU KNOW?

Q. Why is electric current not considered a vector quantity even though it has a direction?

Electric current is not a vector quantity because it does not follow vector addition rules, even though it has a defined direction.





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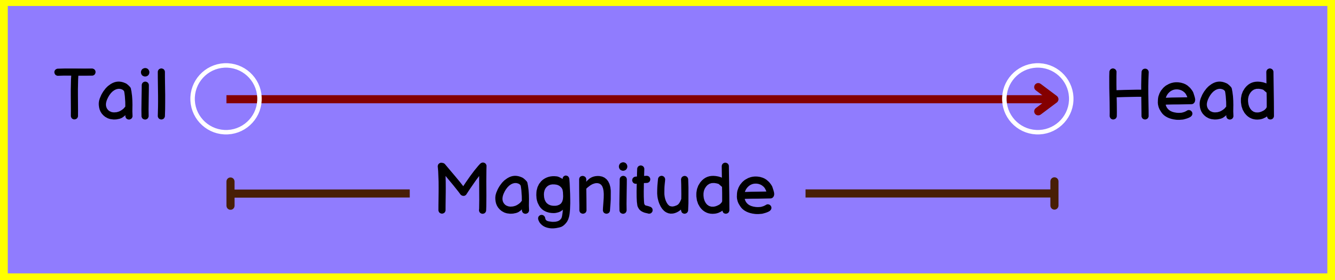
Basics of Vector

Representation of a Vector

Vectors are typically denoted by symbols with an arrow on top. $\vec{A}$

Parts of a Vector:

- **Tail:** Starting point of the vector.
- **Head:** Endpoint showing the direction.



$$\vec{A} = |\vec{A}|\hat{A}$$

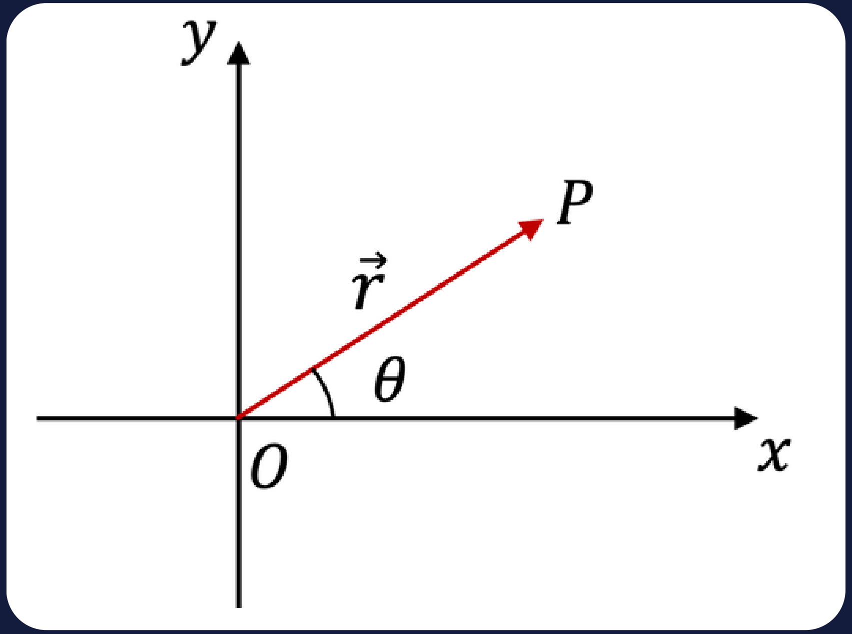
- $|\vec{A}|$: Magnitude (length).
- $\hat{A}$: Unit vector indicating direction.

Direction: Indicated by where the arrow points.

Types of Vector

Position Vector

The position of an object in a plane is described relative to a reference point (origin O).

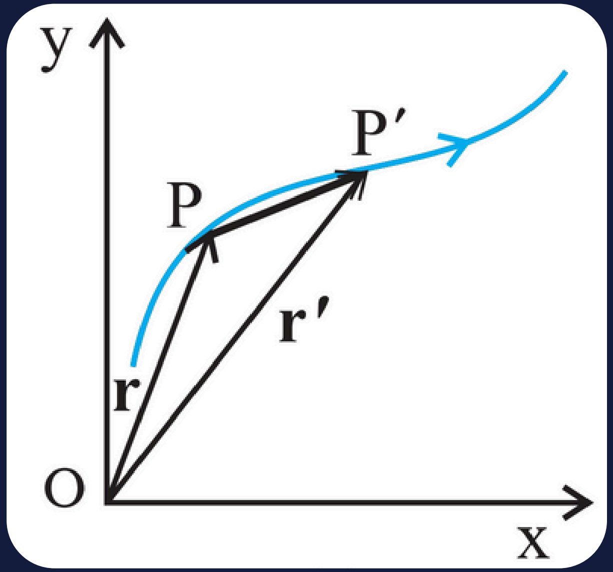


The position vector of point P at time t is written as:

$$\vec{r} = \vec{OP}$$

The length of the vector represents magnitude, and the arrow's direction shows the object's direction from origin.

Displacement Vector



Represents the change in position between two points.

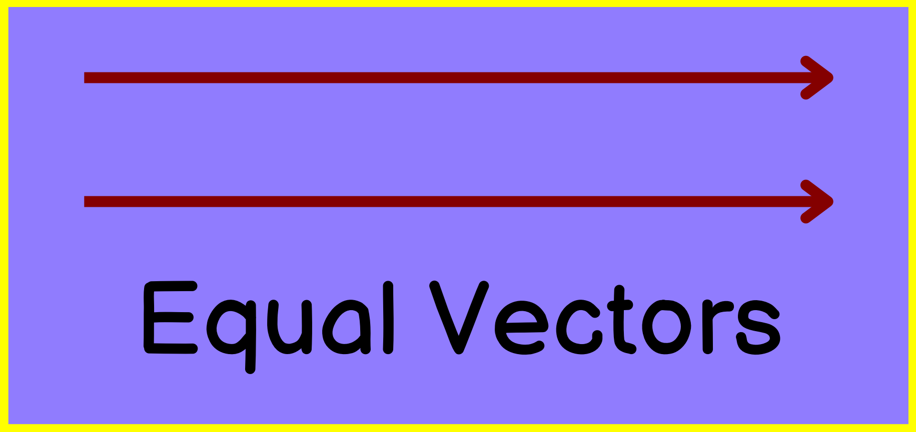
If object moves from point P to P′, the displacement vector is:

$$\vec{PP'} = \vec{r'} - \vec{r}$$

It is shown as a straight arrow from P to P′, regardless of the actual path taken.

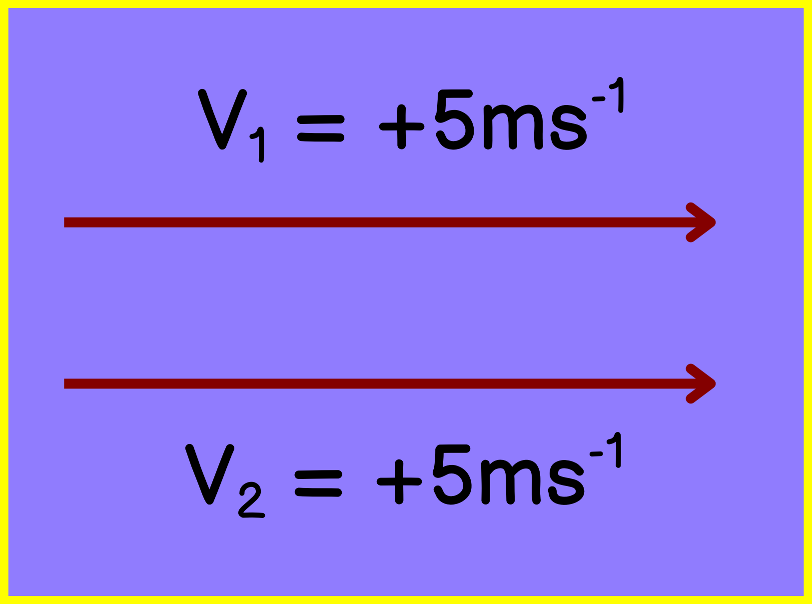
Equal Vectors

Vectors that are identical in magnitude, direction, and of the same type (nature). Such vectors behave the same in vector operations.



Conditions for Equal Vectors:

- Same Magnitude (e.g., 5 ms^{-1} and 5 ms^{-1})
- Same Direction (e.g., both along +x)
- Same Physical Nature (e.g., both are Velocity)

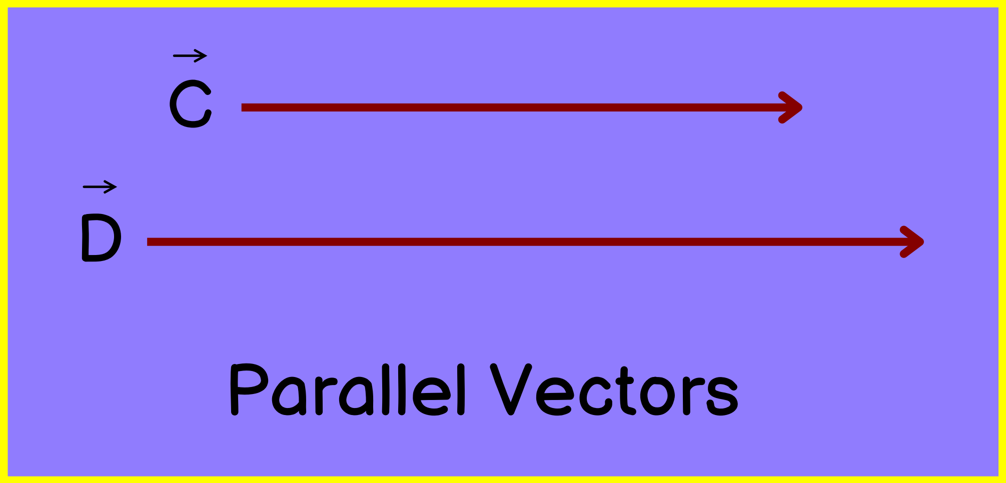


How to Check Equality:

- Shift one vector parallel to itself (without rotation or changing its length).
- If tails coincide and tips coincide, the vectors are equal.

Parallel Vector

Vectors that point in the same direction. Their magnitudes may differ. They lie along the same or parallel lines. Example: $\vec{C} \parallel \vec{D}$

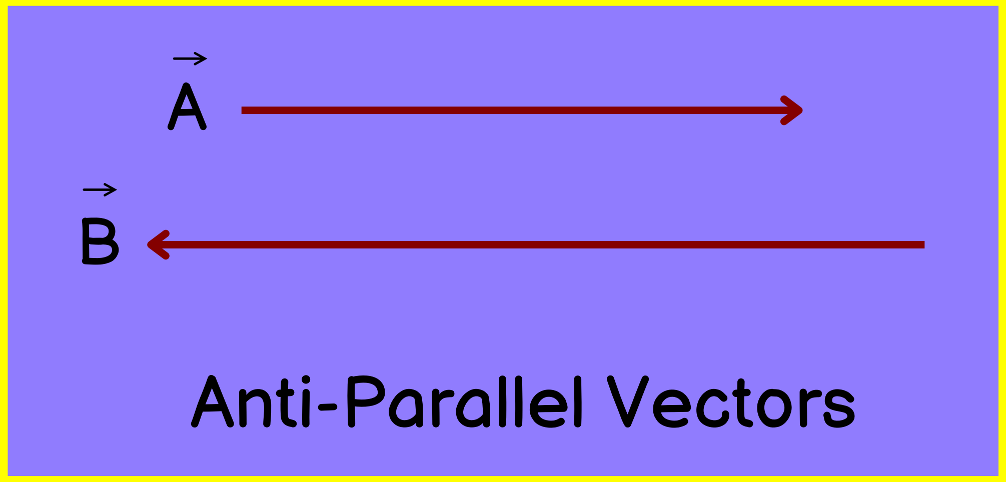


Parallel Vectors

Anti-Parallel Vectors

Vectors that lie on parallel lines but in opposite directions.

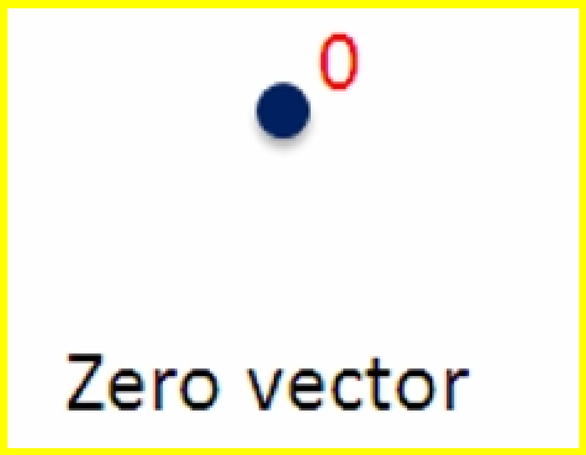
Their magnitudes may differ. Example: $\vec{A} = -\vec{B}$



Anti-Parallel Vectors

Zero Vector

A vector with zero magnitude. Direction is undefined or arbitrary.
Denoted as: $\vec{A} = 0, |\vec{A}| = 0$



Zero vector

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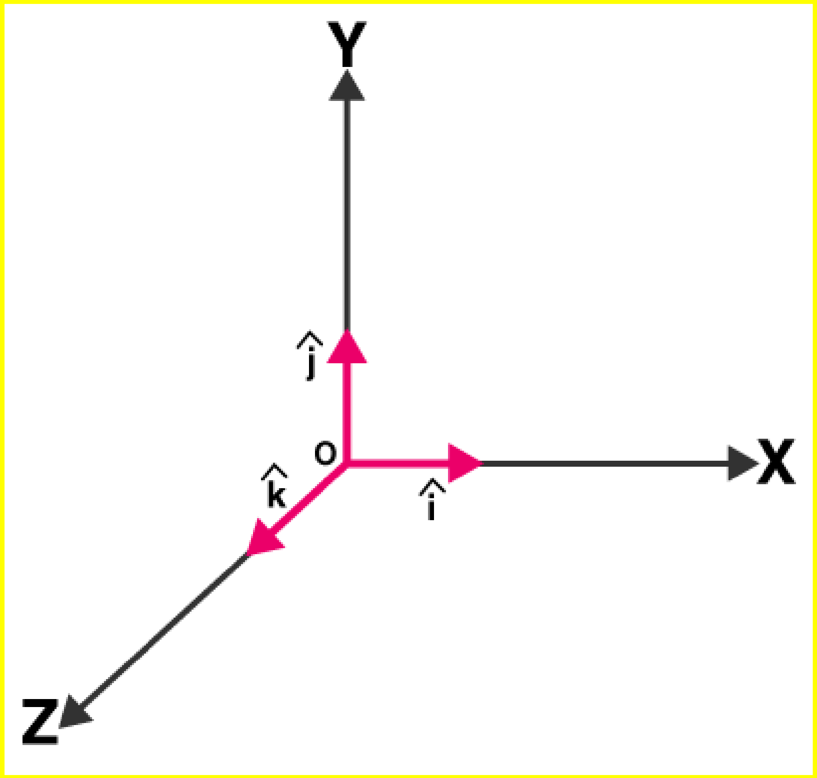
Unit Vectors

A unit vector is a vector that has:

- Magnitude = 1
- A Specific direction

Representation: If A is any vector, its unit vector is: $\hat{A} = \frac{\vec{A}}{|\vec{A}|}$



Unit Vector In 3D

Unit Vector in X - Axis : $\hat{i}$

Unit Vector in Y - Axis : $\hat{j}$

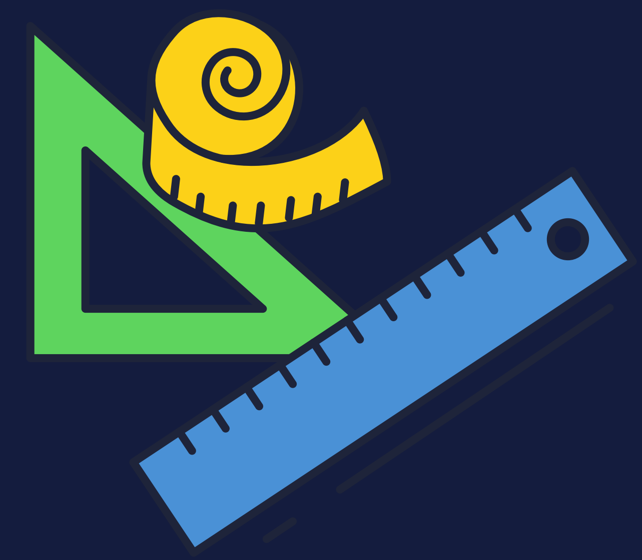
Unit Vector in Z - Axis : $\hat{k}$

Representation of Unit Vector:

$$\vec{A} = |\vec{A}| \cdot \hat{A}$$

Vector = Magnitude × Direction

Q. Find the unit vector in the direction of vector A = $3\hat{i} + 4\hat{j}$



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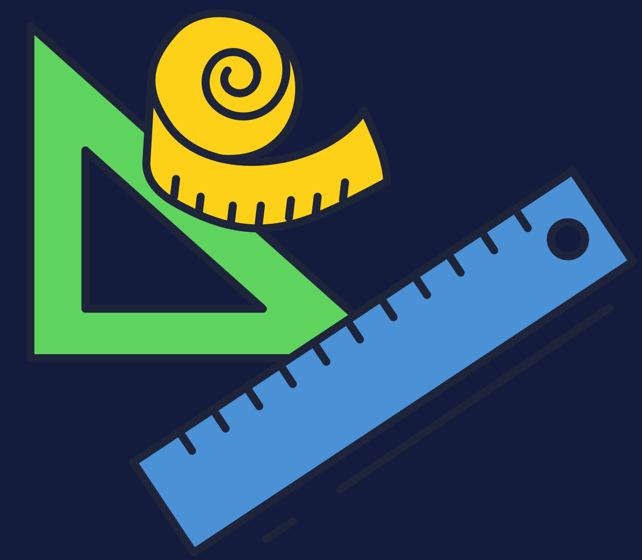
Q. Find a unit vector parallel to the resultant of the vectors
A = $4\hat{i} - \hat{j} + 2\hat{k}$ and B = $-\hat{i} + 2\hat{j} + 3\hat{k}$



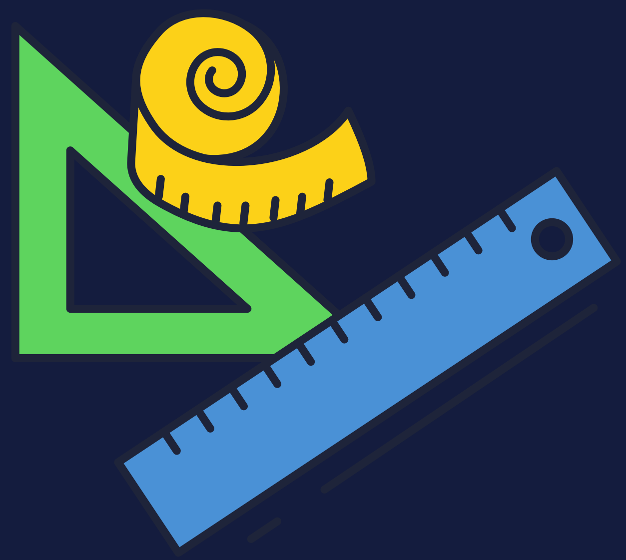
Q Two vectors are said to be equal if:

a) They have the same magnitude

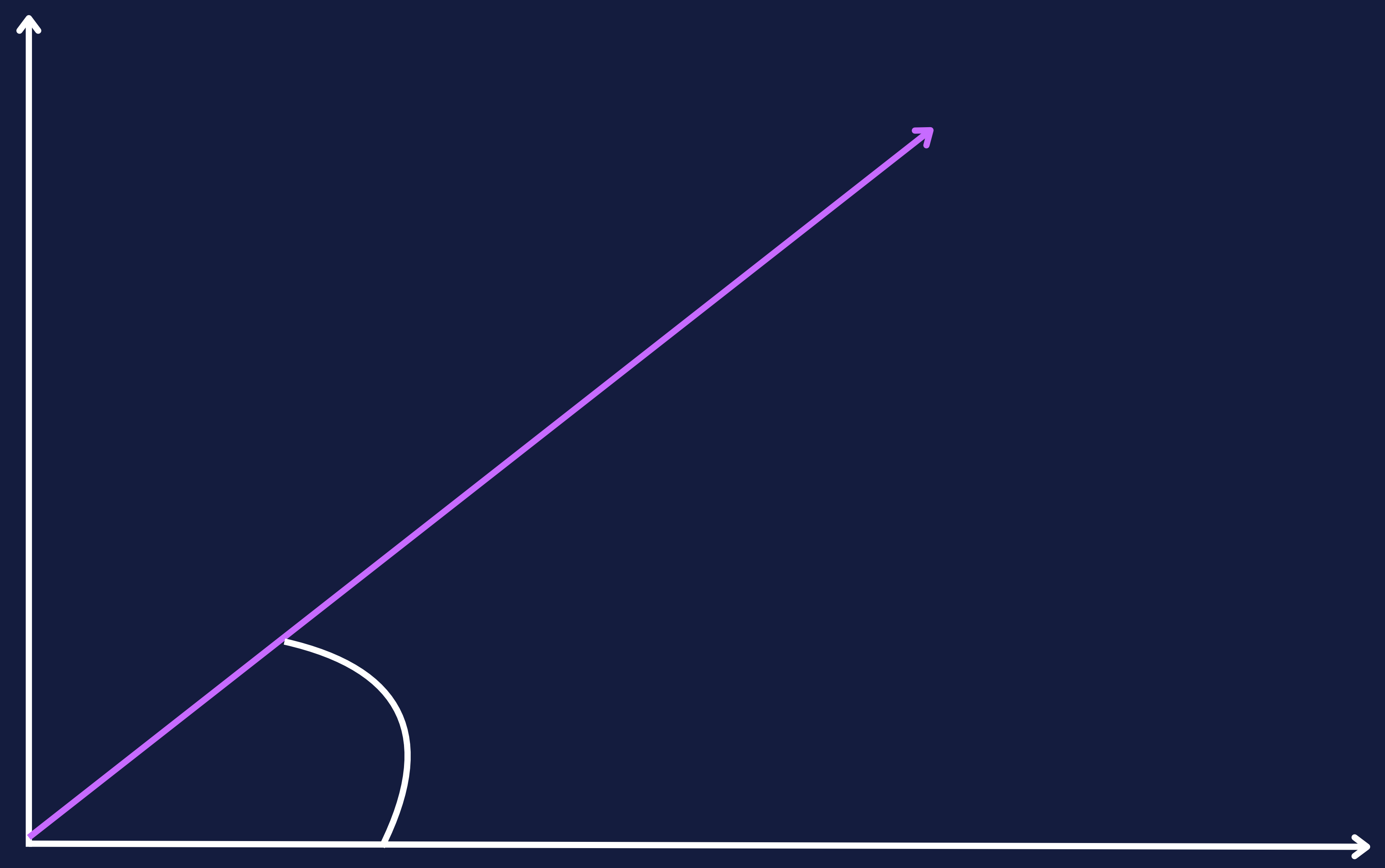
b) They are in the same direction

c) They have the same magnitude and direction

d) Their sum is zero



Angle between Vectors



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Resolution of Vectors

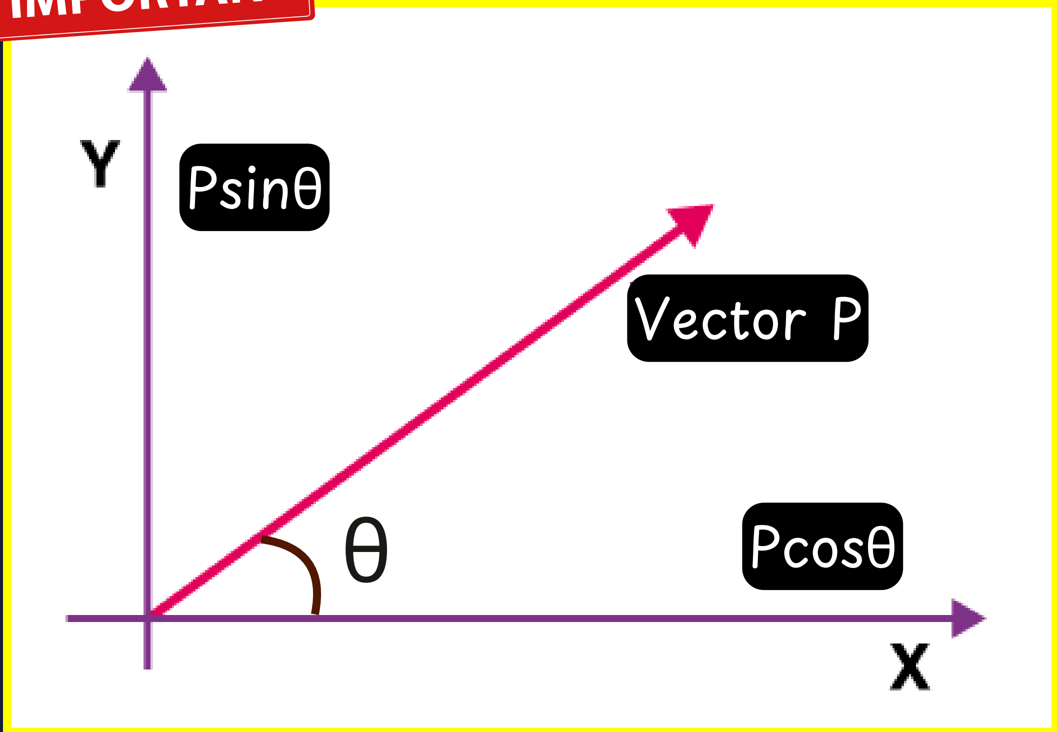
Breaking a vector into its components along the coordinate axes is called the resolution of a vector.

Components of a Vector

Vector P can be resolved as:

- x-component: $P_x = P\cos\theta$
- y-component: $P_y = P\sin\theta$

IMPORTANT



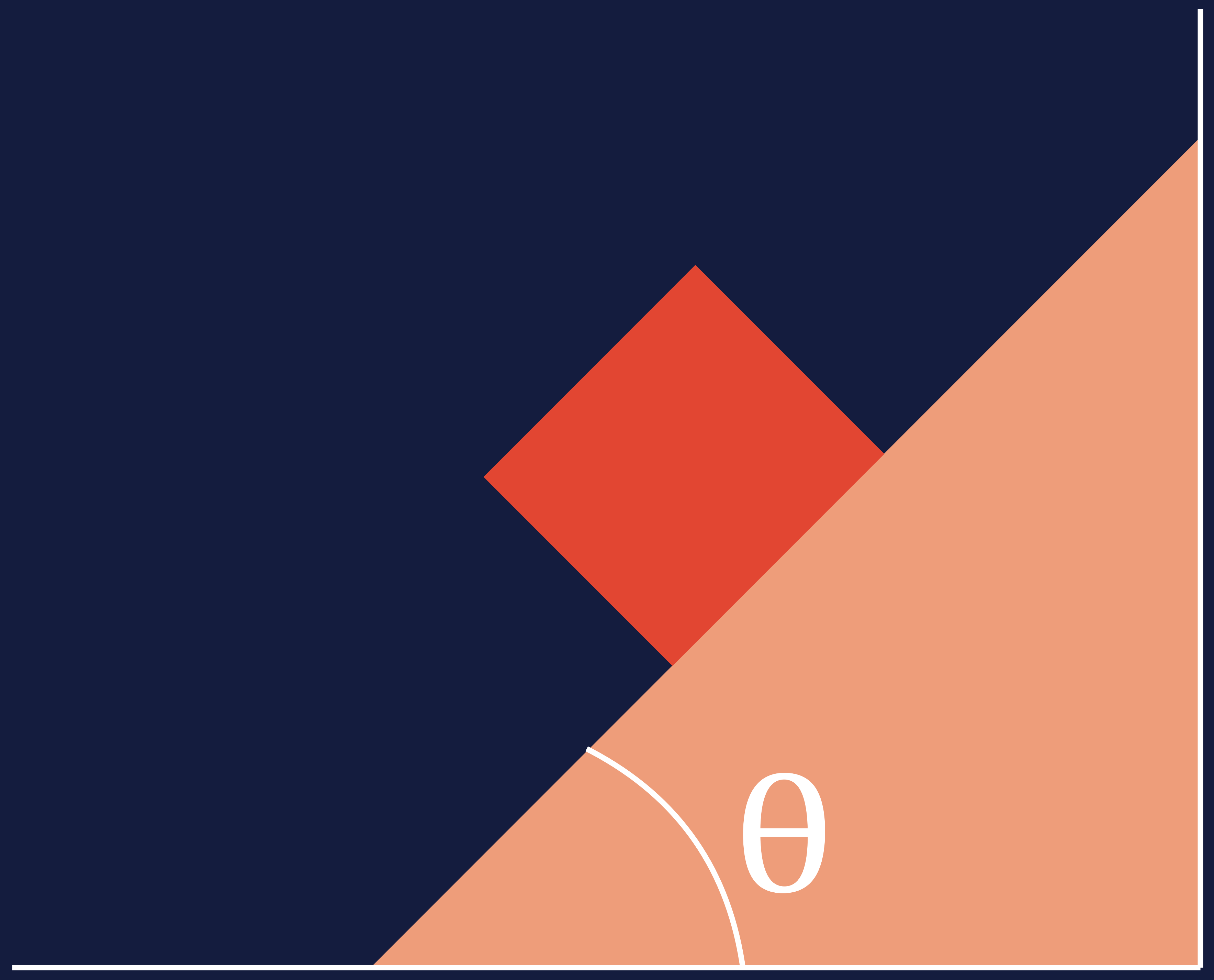
Vector Representation

In 2D: $\vec{R} = x\hat{i} + y\hat{j}$

Magnitude: $|\vec{R}| = \sqrt{x^2 + y^2}$

In 3D: $\vec{R} = x\hat{i} + y\hat{j} + z\hat{k}$

Magnitude: $|\vec{R}| = \sqrt{x^2 + y^2 + z^2}$

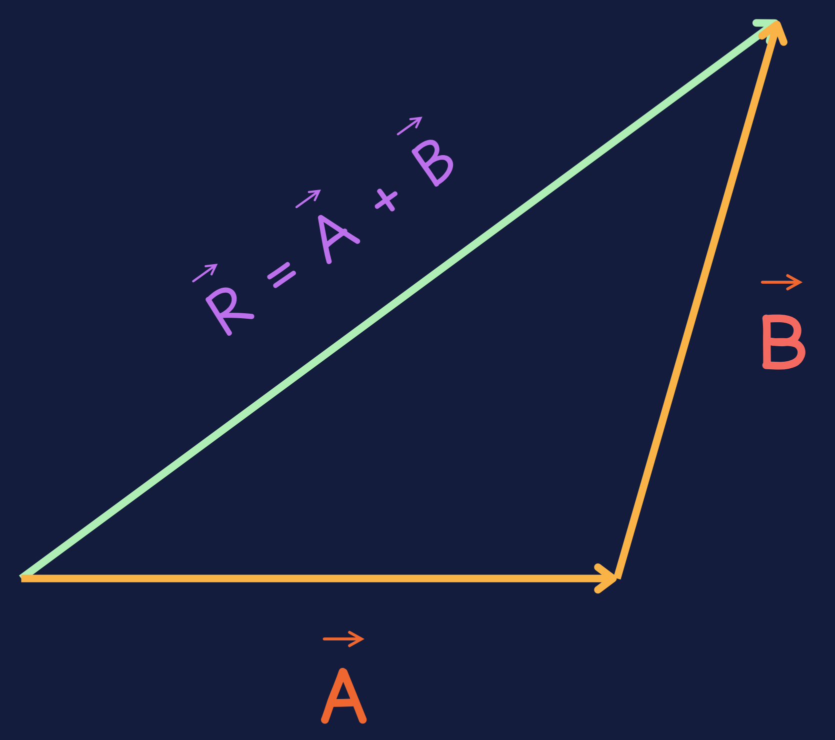

θ

Vector Addition

The **resultant vector** is the combined effect of two or more vectors added together.

Triangle Law Of Vector Addition

If two vectors are placed head-to-tail, the third side (in reverse order) gives the resultant vector, directed from the tail of the first to the head of the second.



$$\vec{R} = \vec{A} + \vec{B}$$

Magnitude of Resultant Vector (R)

If the angle between A and B is θ:

$$|\vec{R}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

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Pythagoras Theorem

Using the identity $\sin^2\theta + \cos^2\theta = 1$:

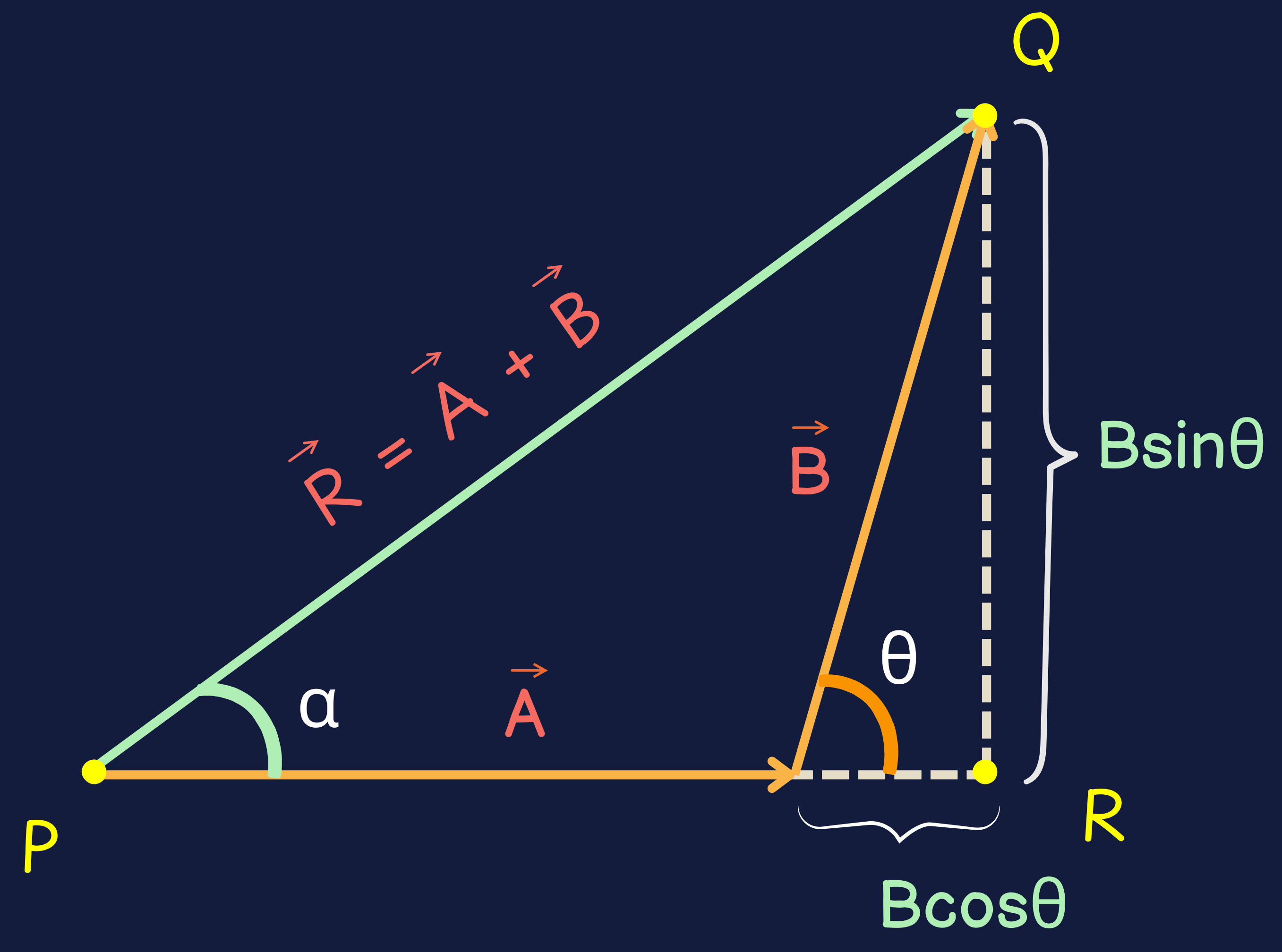
$$R^2 = A^2 + B^2(1) + 2AB\cos\theta$$

Finally, taking square root of both sides:

$$|\vec{R}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

Direction of Resultant Vector: α is the angle between R and the x-axis or the direction of A.

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$



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Resultant Vector

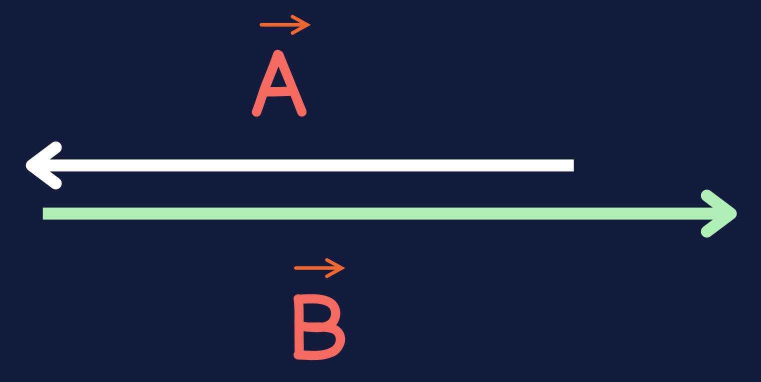
Maximum and Minimum Resultant

- Maximum Resultant (R_{max}): Occurs when $\theta = 0^\circ$

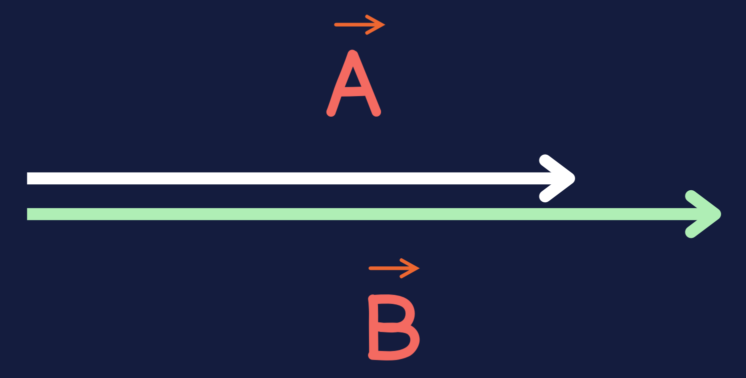
$$R_{max} = A + B$$

- Minimum Resultant (R_{min}): Occurs when $\theta = 180^\circ$

$$R_{min} = |A - B|$$



Minimum Value ($\theta = 180^\circ$)



Maximum Value ($\theta = 0^\circ$)

Range of Resultant

$$R \in [|A - B|, A + B]$$

Magnitude of Resultant Vector

Case 1: Maximum Resultant (θ = 0˚)

Both vectors are in the same direction

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$R = \sqrt{A^2 + B^2 + 2AB(1)} \qquad \{ \cos 0^\circ = 1 \}$$

$$R = \sqrt{(A+B)^2} = A + B$$

Maximum Resultant:

$$R_{max} = A + B$$

Case 2: Minimum Resultant (θ = 180˚)

Both vectors are in the Opposite direction

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$R = \sqrt{A^2 + B^2 + 2AB(-1)}$$ { cos180° = - 1 }

$$R = \sqrt{A^2 + B^2 - 2AB} = \sqrt{(A - B)^2}$$

Minimum Resultant:

$$R_{min} = |A - B|$$

Case 3: Perpendicular Vectors (θ = 90°)

When vectors are at right angles:

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$R = \sqrt{A^2 + B^2 + 0}$$ { cos90° = 0 }

$$R = \sqrt{A^2 + B^2}$$

Pythagorean Relation:

$$R^2 = A^2 + B^2$$

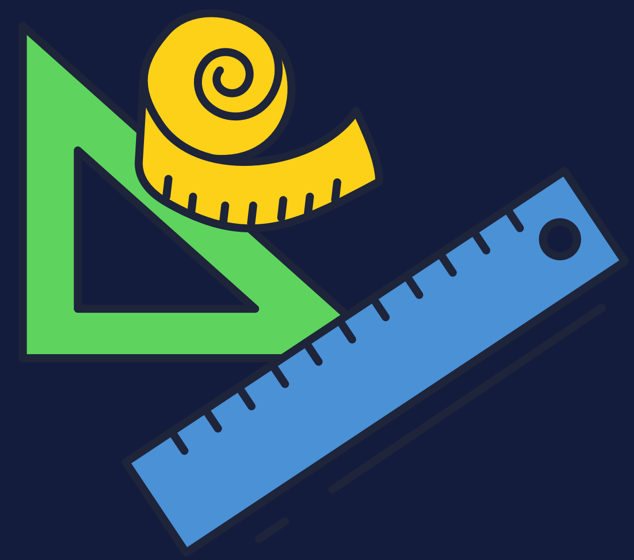
Q . For two vectors A and B, the resultant is maximum when the angle between them is:

(a) 0°

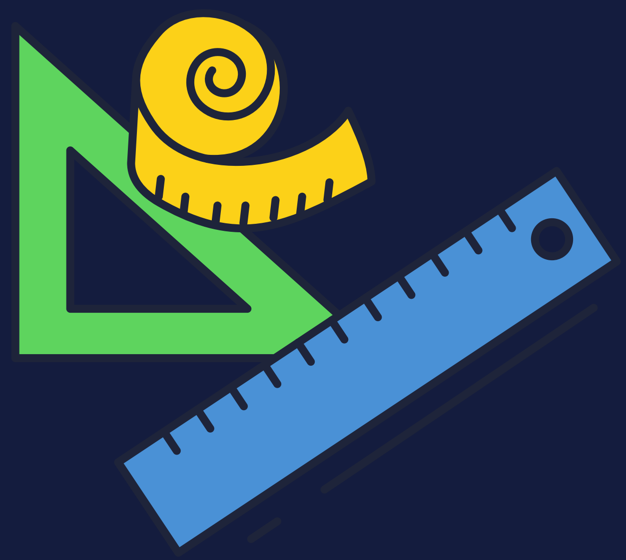
(b) 90°

(c) 180°

(d) 120°



Q Two vectors A = 5 N and B = 5√3 N act at 90° to each other. What is the angle between vector A and the resultant vector R?



Q. Two vectors, both equal in magnitude, have their resultant equal in magnitude to either of them. Find the angle between the two vectors.

NEET 2017

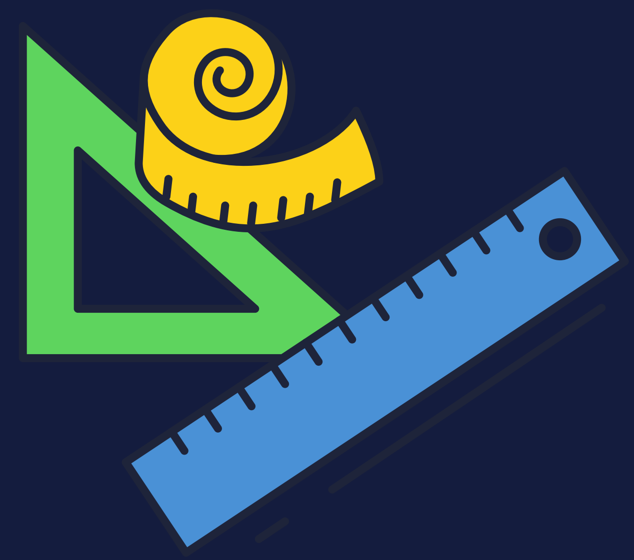
Q Two forces of magnitude 5 N and 10 N act at an angle of 120°. What is the magnitude of the resultant force?

A) 5 N

B) 10 N

C) 15 N

D) 8.66 N



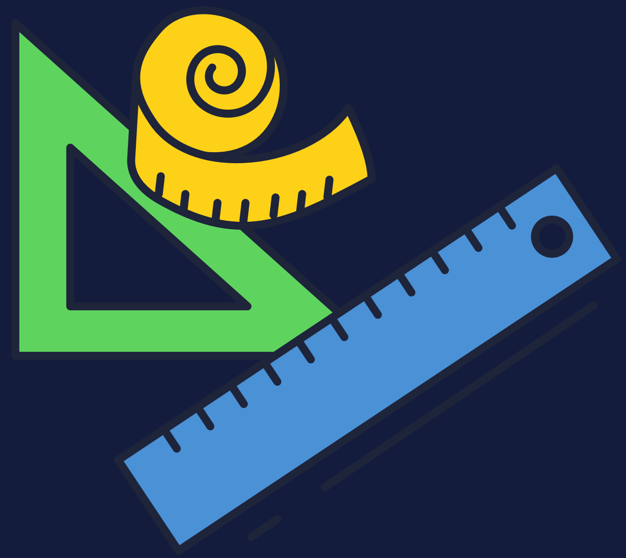
Q. Two vectors have the same magnitude. If the magnitude of their sum is equal to the magnitude of their difference, what is the angle between them?

(a) 90°

(b) 180°

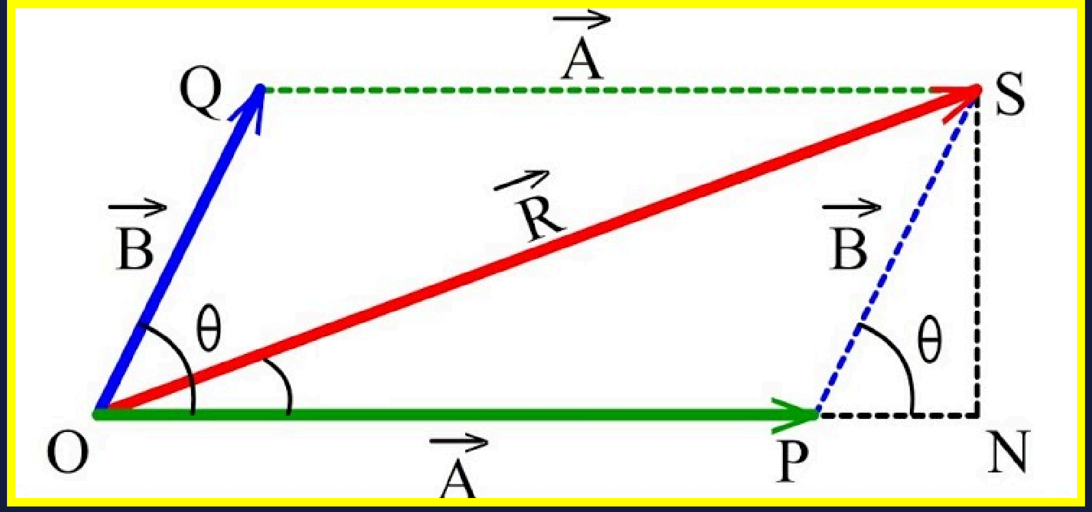
(c) 45°

(d) 60°



Parallelogram Law Of vector addition

When two vectors are shown in both direction and size as the two adjacent sides of a parallelogram, the diagonal beginning from the same point represents the resultant or sum of these vectors.

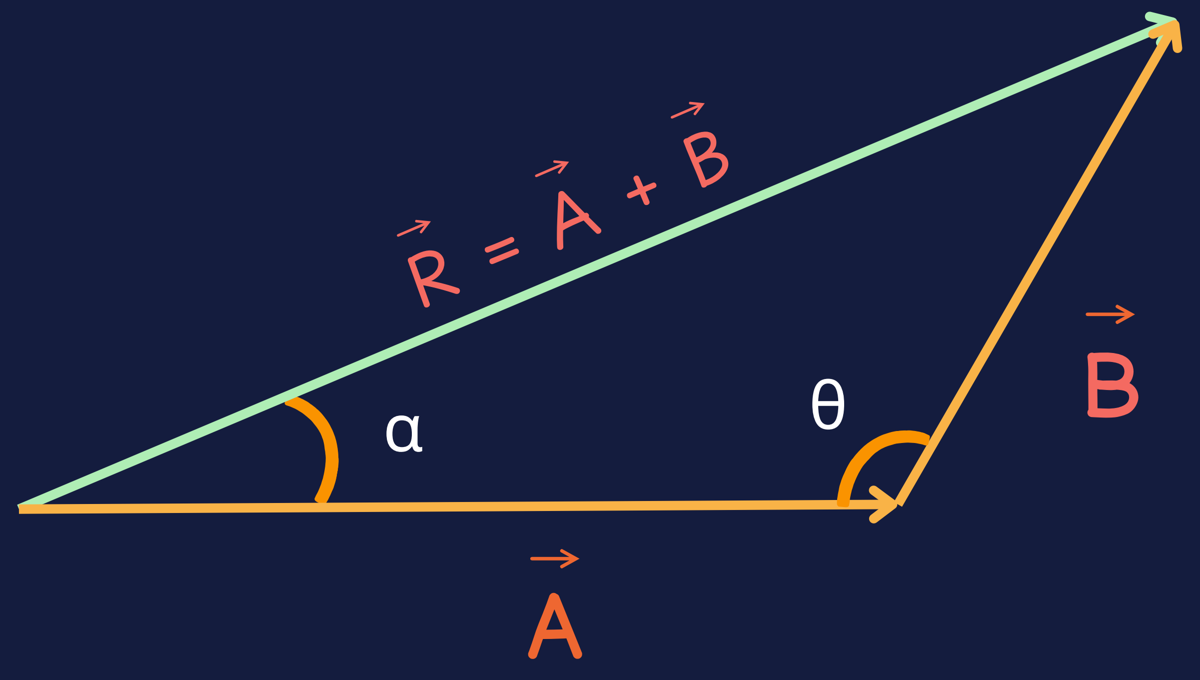


Resultant Magnitude:

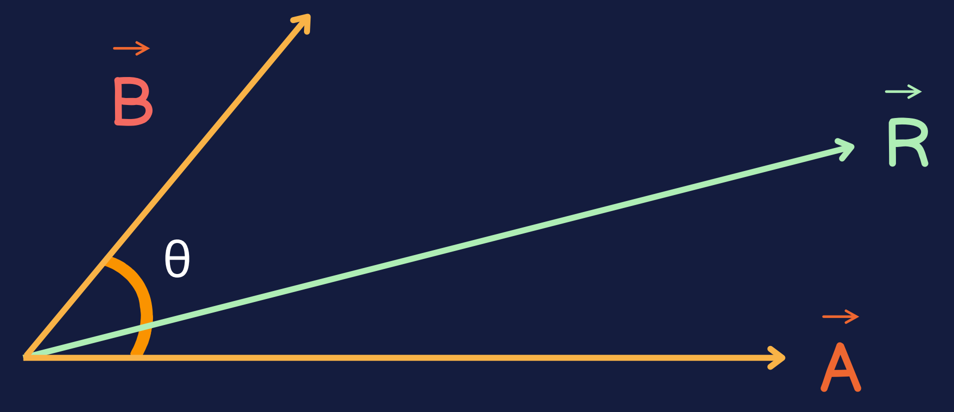
$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

Direction (Angle α with vector A):

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$

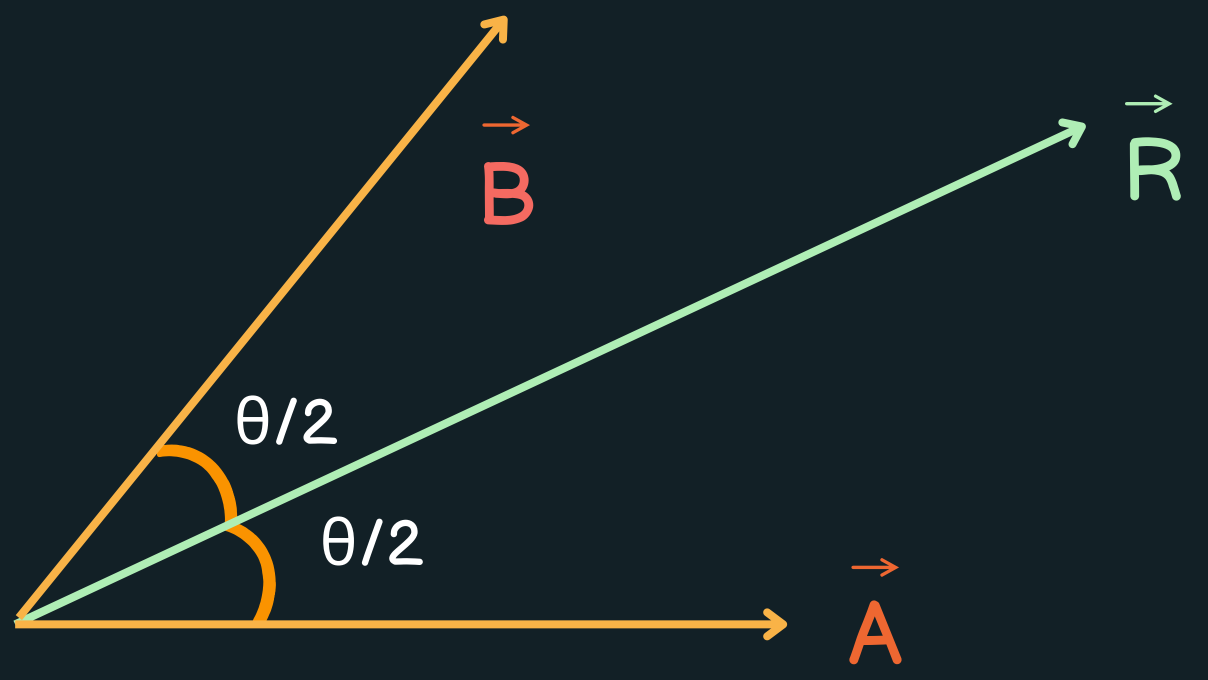


The resultant vector is tilted towards the larger vector.



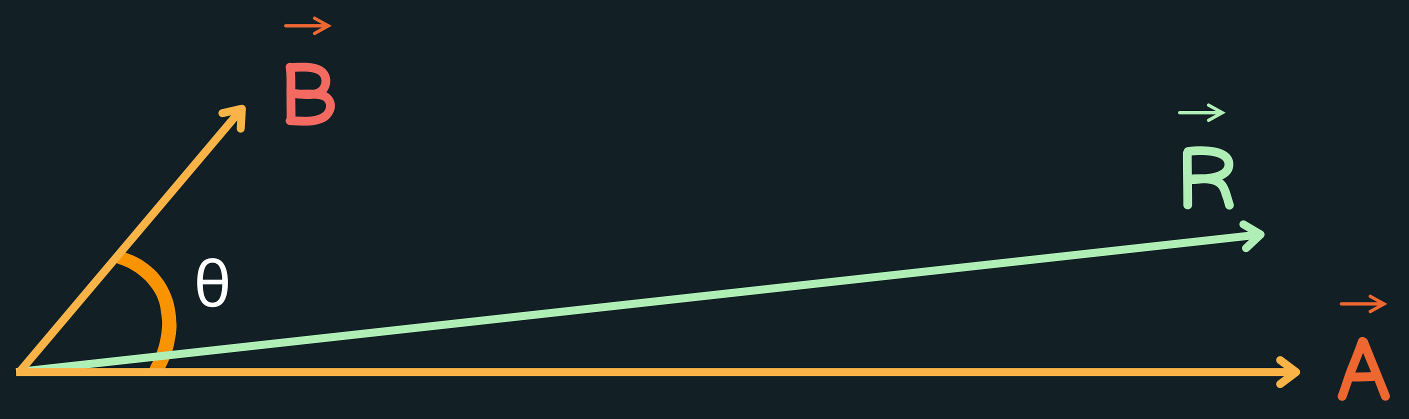
When vectors are equal in magnitude:

Angle between vectors = θ ⇒ resultant splits θ equally.

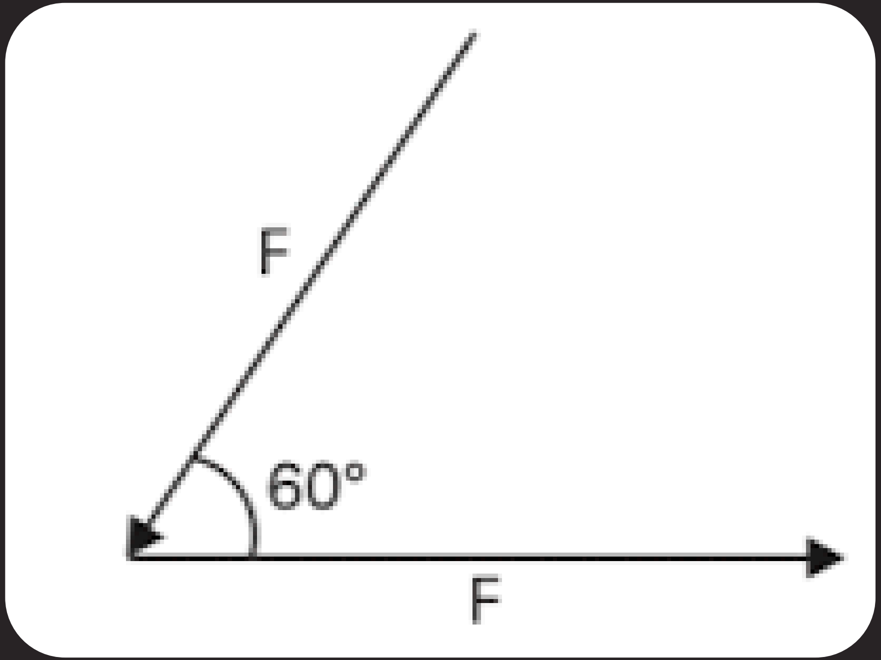


In case of very unequal vectors:

- Resultant vector almost aligns with the larger vector.



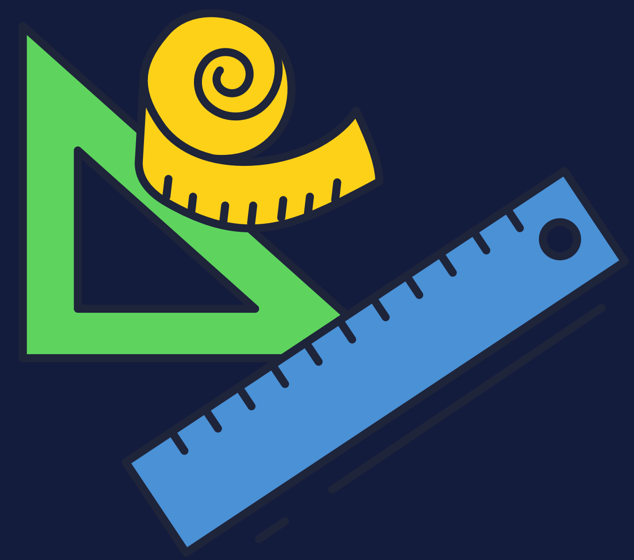
Q. Two forces, each equal to F act the shown figure. Their resultant is:



(a) F/ 2
(b) F
(c) √3 F
(d) √5 F

Q NEET 2020:

Resultant of two vectors of equal magnitude at 120° is $\sqrt{3}$ times each. What is the angle between them?



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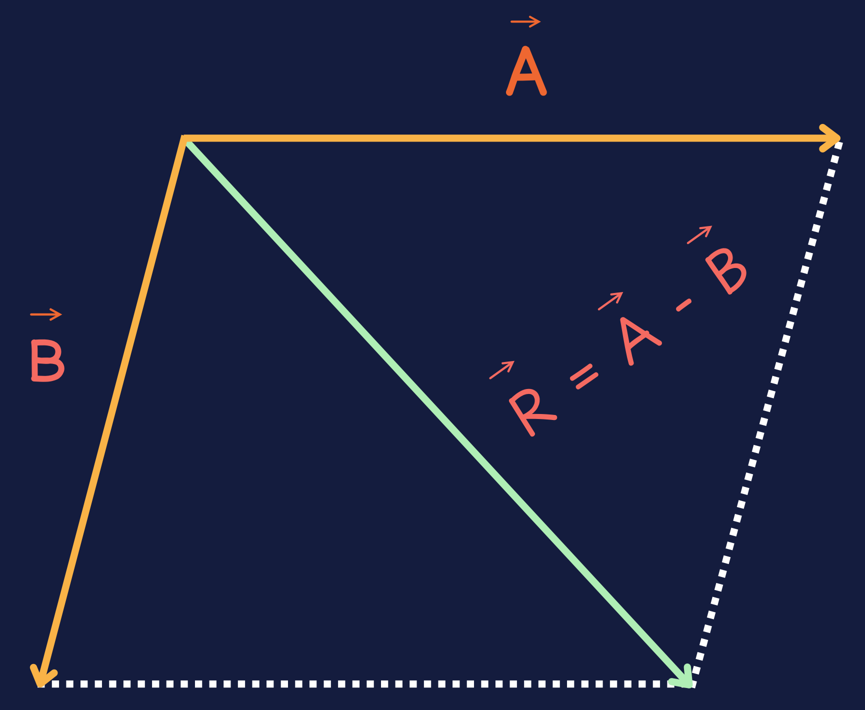
Subtraction of Vectors

Subtraction means adding the reverse direction of vector B to vector A.

$$\vec{R} = \vec{A} - \vec{B}$$

$$\vec{R} = \vec{A} + (-\vec{B})$$



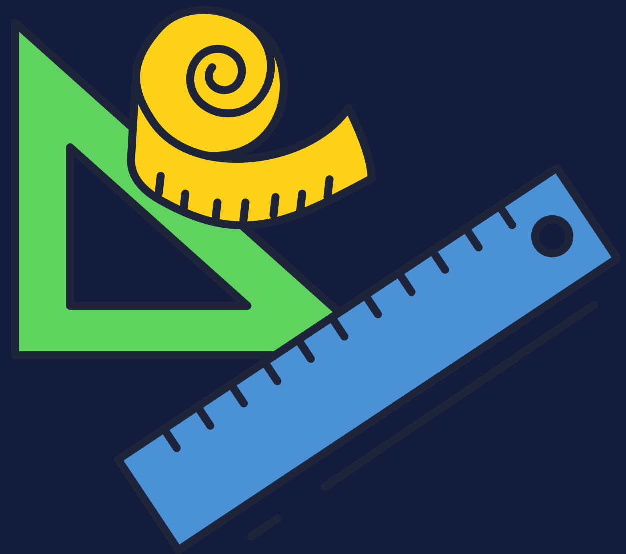
Resultant Vector Magnitude (R)

Addition: $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

Subtraction: $R' = \sqrt{A^2 + B^2 - 2AB\cos\theta}$

The cos term becomes negative in subtraction because vector B is replaced by −B, which flips the direction.

Q. If $A = 3\hat{i} + 4\hat{j} + 2\hat{k}$ and $B = \hat{i} + 2\hat{j} + \hat{k}$, then find the magnitude of $|A - B|$.



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Motion in 2-Dimensions

- 2D motion = combination of two independent 1D motions along x- and y-axes.
- Net motion = vector sum of x and y motions.
- Both motions share the same time (t).

If a particle moves in the x-y plane with position vector: r(t) = x(t) î + y(t) ĵ

Velocity in 2D

$$\vec{v}(t) = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$$

$$\vec{v}(t) = v_x\hat{i} + v_y\hat{j}$$

Acceleration in 2D

$$\vec{a}(t) = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j}$$

$$\vec{a}(t) = a_x\hat{i} + a_y\hat{j}$$

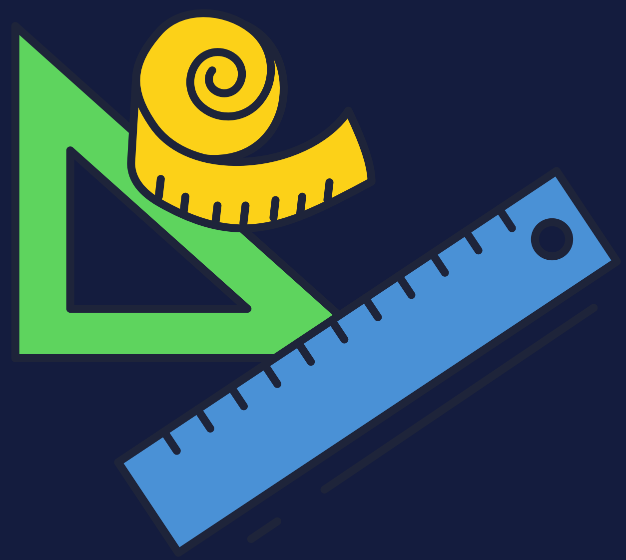
Q. A particle moves in the x-y plane such that its position at time t is given by

$$\vec{r}(t) = 3t^2\hat{i} + 2t\hat{j}$$

Find:

(a) The velocity vector at t=2s

(b) The acceleration vector



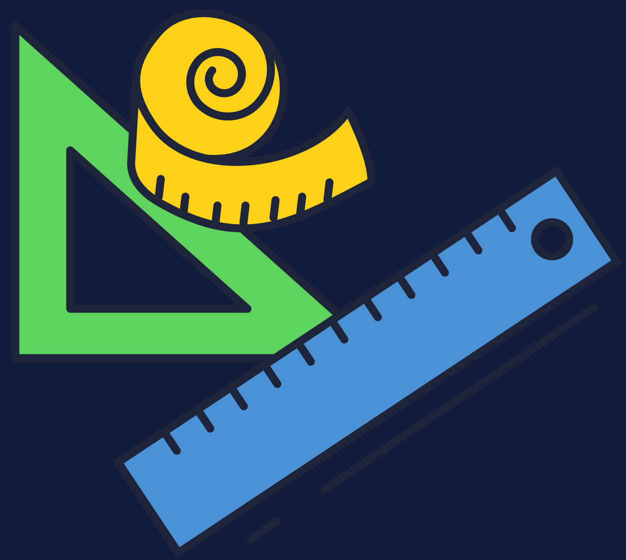
Q. A particle has an initial velocity $\vec{u} = 2\hat{i} + 5\hat{j}$ m/s and acceleration $\vec{a} = 0.3\hat{i} + 0.2\hat{j}$ m/s^2. Find the magnitude of velocity after 8 seconds.

A. 8 units

B. 9.2 units

C. $7\sqrt{2}$ units

D. 10.1 units



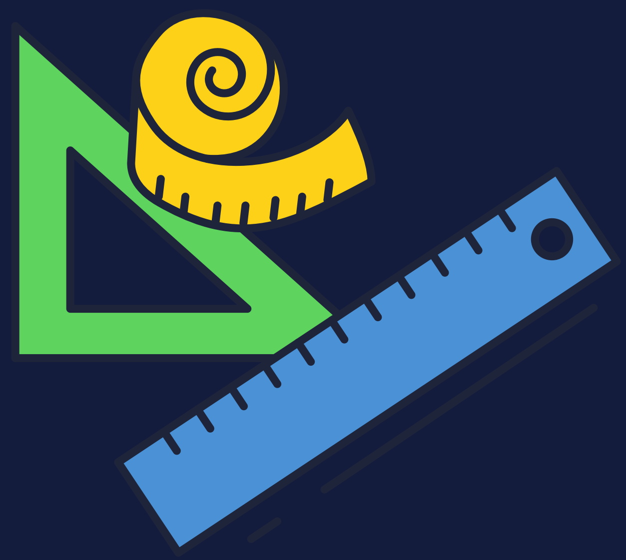
Q. A particle moves such that $x = \alpha t^3$ and $y = \beta t^3$, What is the magnitude of its velocity vector at time t?

(a) $3t\sqrt{(\alpha^2 + \beta^2)}$

(b) $3t^2\sqrt{(\alpha^2 + \beta^2)}$

(c) $\sqrt{(\alpha^2 + \beta^2)}$

(d) $t^2\sqrt{(\alpha^2 + \beta^2)}$



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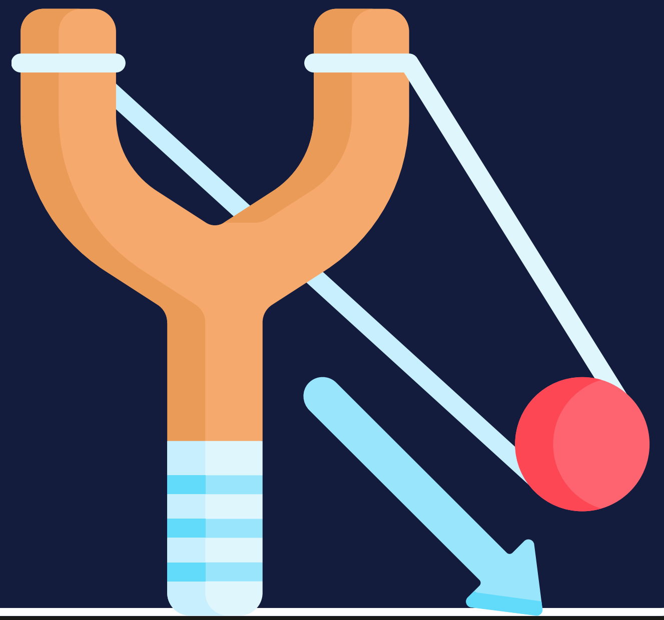
Projectile Motion



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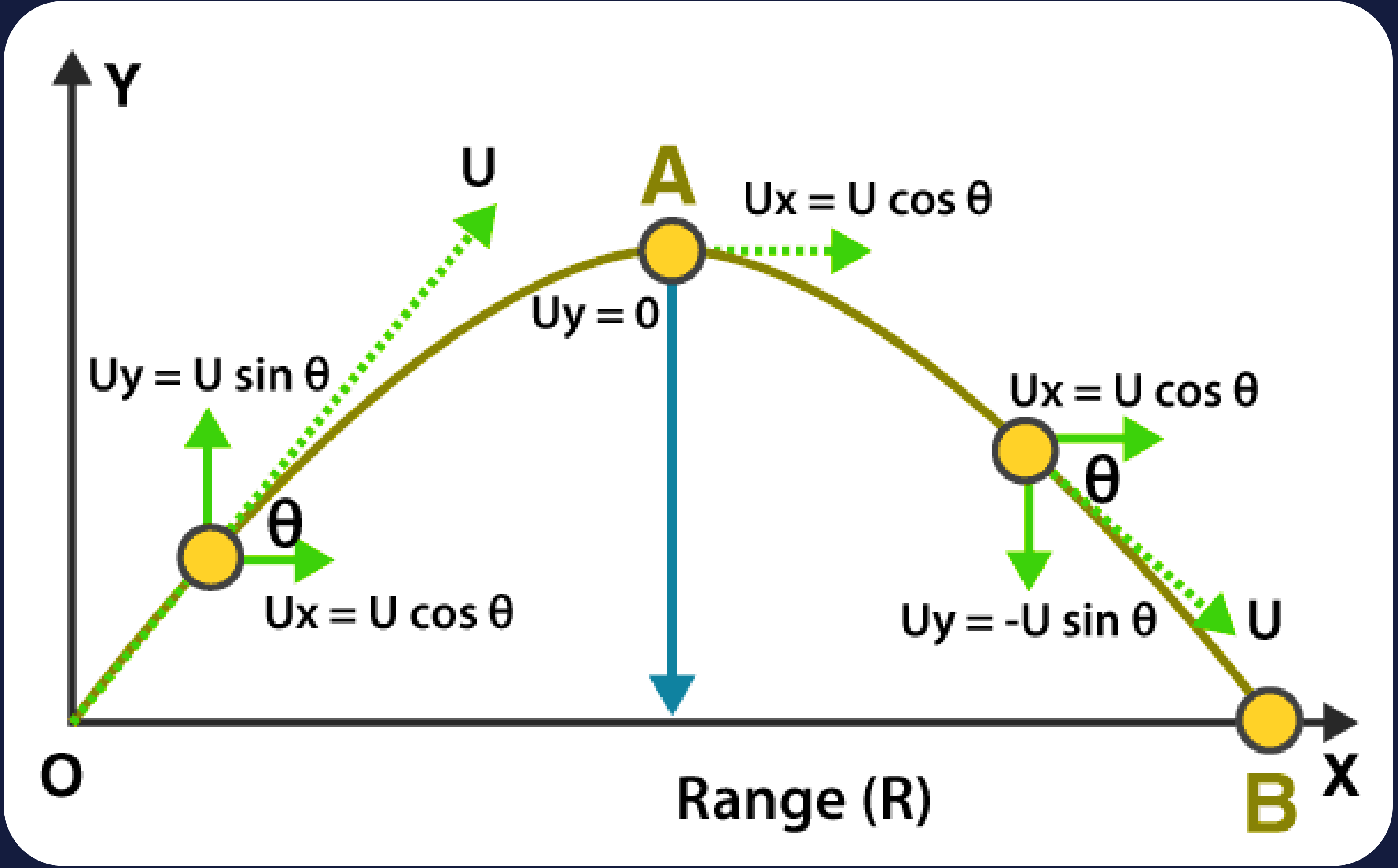
Projectile Motion

*"When an object is projected with some initial velocity at an angle to the horizontal, and the only force acting on it (after projection) is gravity, it undergoes **projectile motion.**"*

- *It is a two-dimensional motion (in both x and y directions).*
- *The horizontal component of velocity remains constant (no acceleration).*
- *The vertical component of velocity changes due to acceleration due to gravity (g).*
- *The path followed is a parabola.*



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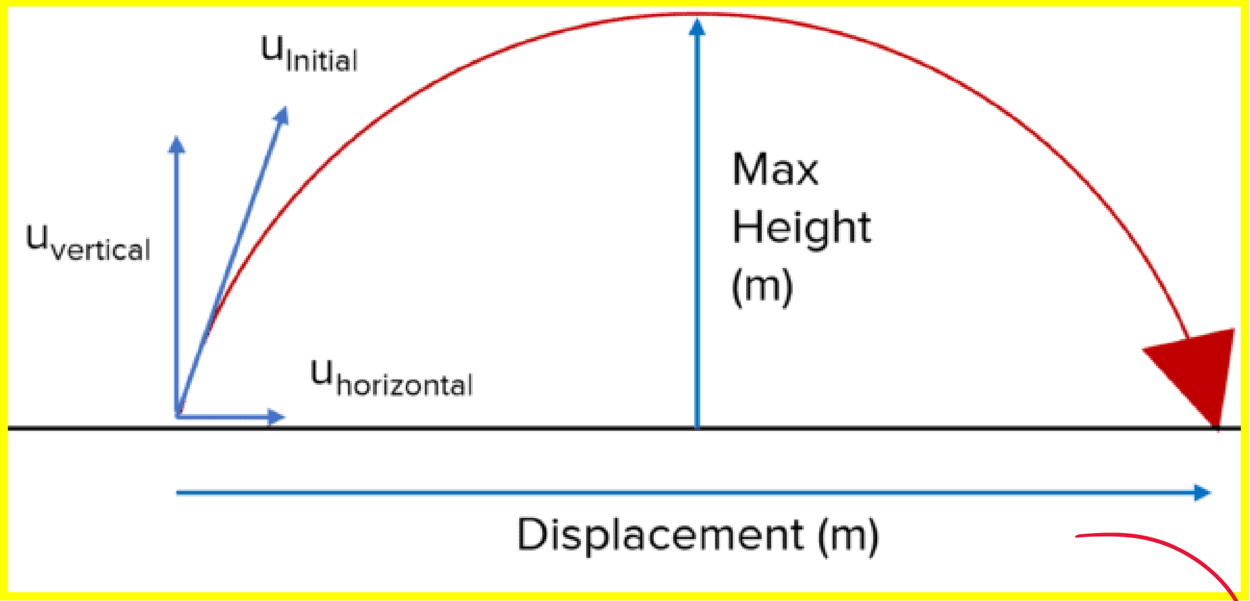
2-D Projectile Motion

- Components of initial velocity:
  - $U_x = U\cos\theta$
  - $U_y = U\sin\theta$
- Point of projection is the origin (0,0).

- Initial velocity: U
- Angle of projection: θ
- R: Range
- H_{max}: Maximum height
- T: Time of flight
- Horizontal Range: $S_x = R$



Curve's Tangent = Direction of Velocity

X-Direction (Horizontal Motion):

Displacement: S_x

Velocity: $U_x = U\cos\theta$

Acceleration: $a_x = 0$

Uniform Motion:

- $V_x = U_x = \text{constant}$
- $S_x = U_x t + \frac{1}{2} a_x t^2 = U_x t$
- $U_x = S_x / t$

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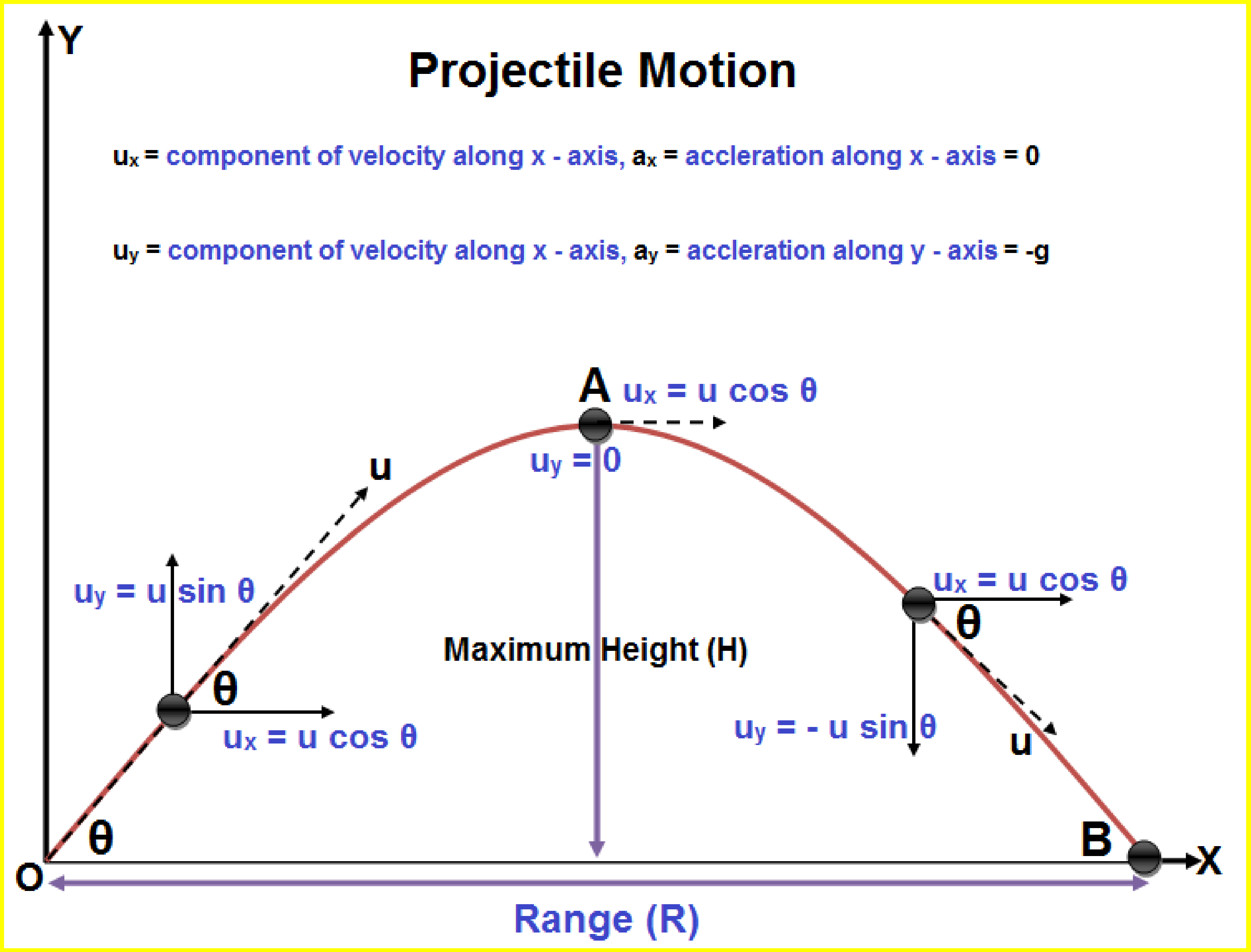
Y-Direction (Vertical Motion):

Initial velocity: $U_y = U\sin\theta$

Acceleration: $a_y = -g$ (downward)

Equations:

- $V_y = U_y + a_y t = U_y - gt$
- $S_y = U_y t + \frac{1}{2} a_y t^2 = U_y t - \frac{1}{2} gt^2$
- $V_y^2 = U_y^2 + 2a_y S_y = U_y^2 - 2g S_y$
- $S_y / t = (U_y + V_y) / 2$



Time of Flight (T)

Time of flight is the total duration a projectile remains in the air, from the instant it is launched until it returns to the same vertical level from which it was projected.

Given: $a_y = -g$

Initial vertical velocity: $U_y = U\sin\theta$

Final vertical velocity at highest point: $V_y = 0$

- Using the equation: $V_y = U_y + a_y t$

$$0 = U\sin\theta - gt$$

$$t = U\sin\theta / g$$

y-direction Analysis

$$T = 2t$$

$$T = \frac{2U\sin\theta}{g}$$

Special Case: $\theta = 90^\circ$ (This becomes the 1D Projectile or Free Fall case.)

$$T = \frac{2U\sin\theta}{g} \Rightarrow T = \frac{2U\sin 90^\circ}{g} = \frac{2U}{g}$$

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Maximum Height (H_{max})

y-direction Analysis

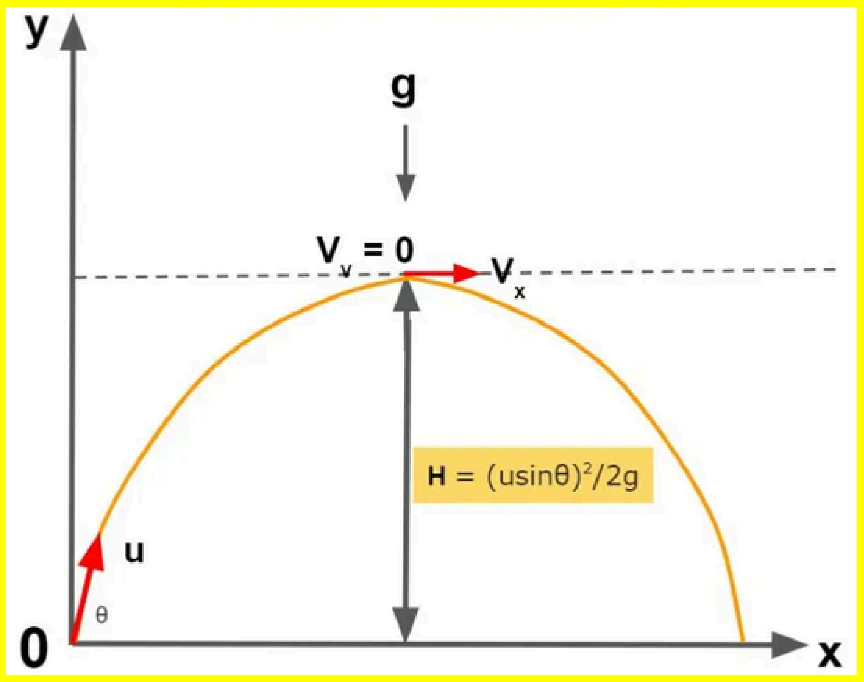
Maximum height is the highest vertical position reached by a projectile during its trajectory.

Given: V = 0 (at highest point)

$U_y = U\sin\theta$

$a_y = -g$

$S = H_{max}$



- Using the equation: $V_y^2 = U_y^2 + 2a_y S_y$

$0 = (U\sin\theta)^2 - 2gH_{max}$

$H_{max} = U^2 \sin\theta^2 / 2g$

$$H_{\max} = \frac{U^2 \sin^2\theta}{2g}$$

Horizontal Range or Range

Horizontal range (often simply called "range") is the total horizontal distance a projectile travels before returning to its original vertical position (usually the ground), assuming it is launched and lands at the same height.

Key Concept: Uniform Motion in horizontal direction

Horizontal acceleration: $a_x = 0$

Horizontal Velocity Component: $U_x = U\cos\theta$ = constant

x-direction

- Using the equation: $S_x = U_x t + \frac{1}{2} a_x t^2$

$S_x = U_x T + \frac{1}{2}(0) T^2$

$R = U\cos\theta \cdot T$

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From vertical motion: $T = \dfrac{2U}{g}$

Putting it all together: R = Ucosθ · T

$$\Rightarrow \quad R = U\cos\theta \cdot \frac{2U\sin\theta}{g} = \frac{U^2\sin 2\theta}{g}$$ (2sinθcosθ = sin2θ)

Maximum Range

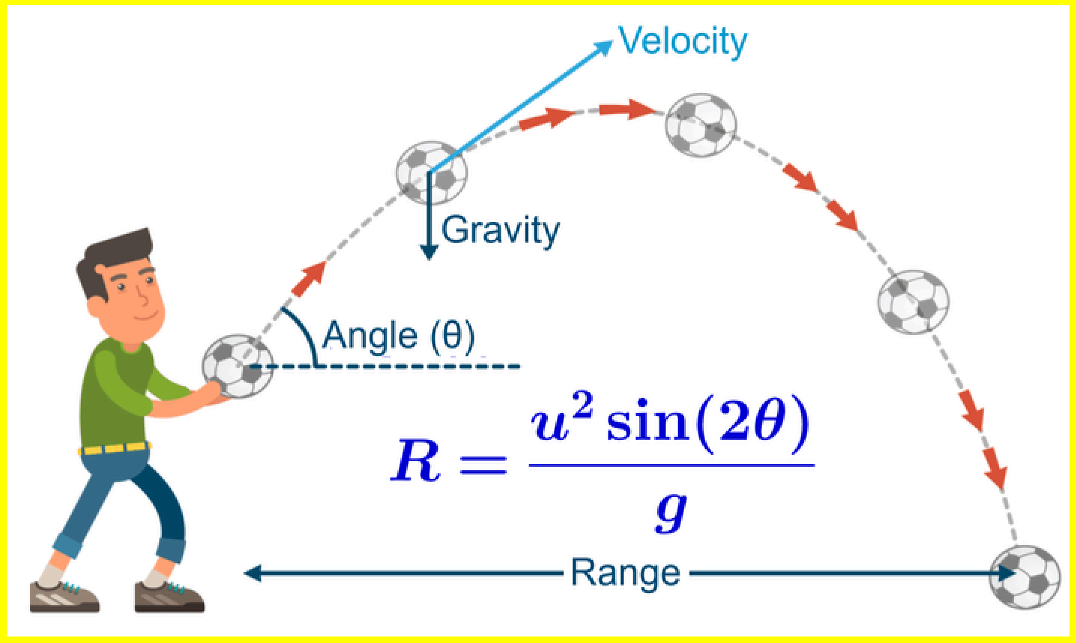
Range Formula: $R = \dfrac{U^2\sin 2\theta}{g}$

Maximizing the value: Maximum value of sin2θ = 1

$\sin 2\theta = \sin 90^\circ$

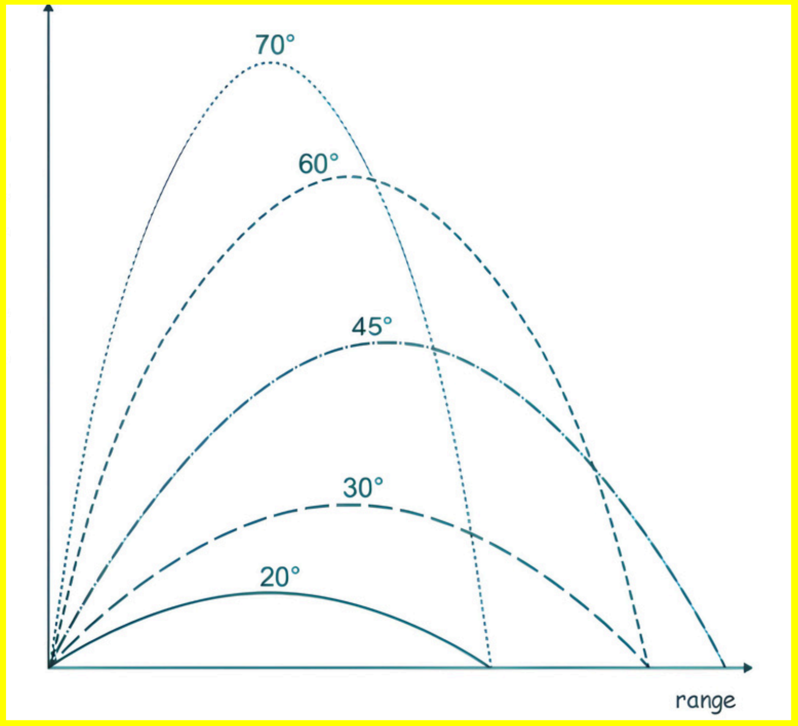
$2\theta = 90^\circ$

$\theta = 45^\circ$ ⟶ Max Range



Same Range

Two projectiles having complementary angles ($\theta_1 + \theta_2 = 90°$) when projected from the same point will cover the same horizontal range.



Given: $\theta_1 = \theta$ and $\theta_2 = 90° - \theta$

Initial velocity = u

Range formula: $$R = \frac{U^2 \sin 2\theta}{g}$$

Proof: For $\theta_1 = \theta$: $$R = \frac{U^2 \sin 2\theta}{g}$$

Proof: For $\theta_2 = 90° - \theta$: $$R' = \frac{u^2 \sin 2(90° - \theta)}{g} = \frac{u^2 \sin(180° - 2\theta)}{g} = \frac{u^2 \sin 2\theta}{g}$$

$\therefore$ $R = R'$, hence proved.

Q . For angles of projection of a projectile at angle $(45° - \theta)$ and $(45° + \theta)$, the horizontal ranges described by the projectile are in the ratio of:

(a) 2 : 1

(b) 1 : 1

(c) 2 : 3

(d) 4 : 1

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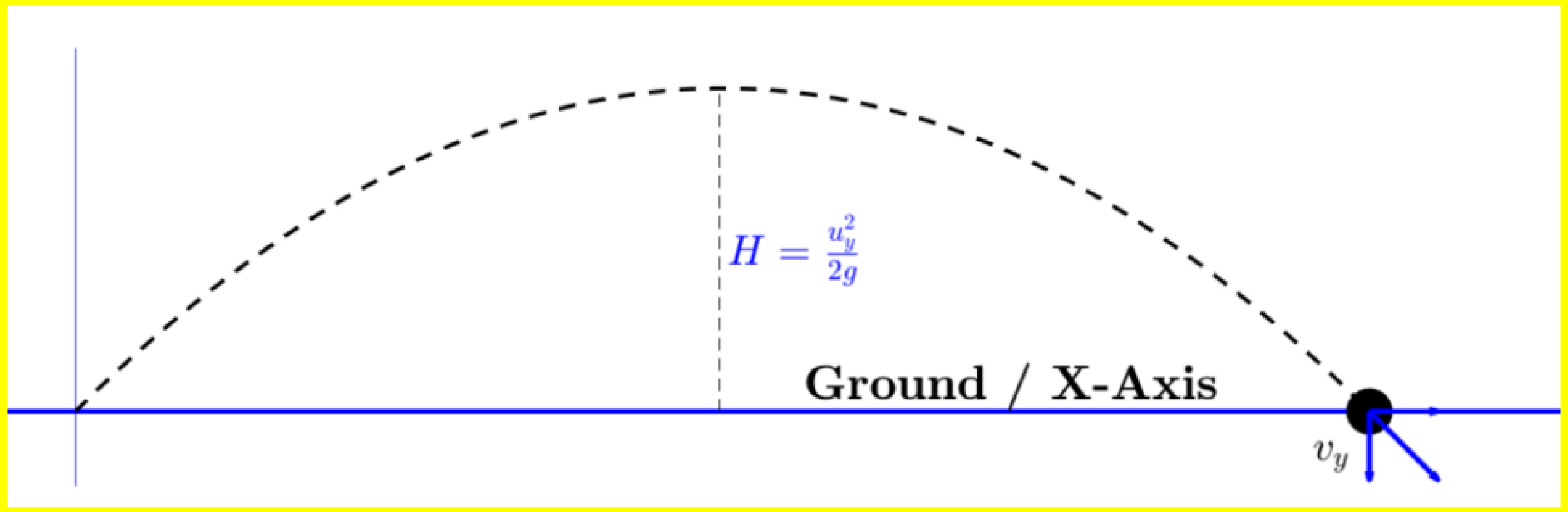
Maximum Possible Height

- Maximum height and range depend on the angle of projection.
- For maximum range, θ = 45°.



Maximum range Formula: $R = \frac{U^2 \sin 2\theta}{g}$

∴ Maximum range at θ = 45°: $R_{\max} = \frac{u^2}{g}$

Maximum Height Formula: $H_{\max} = \frac{U^2 \sin^2 \theta}{2g}$

∴ Maximum Height at θ = 90°: $H_{\max} = \frac{u^2}{2g}$

The Relationship between H_{max} and R_{max} :

$$\frac{H_{\max}}{R_{\max}} = \frac{\frac{u^2}{2g}}{\frac{u^2}{g}} = \frac{1}{2}$$

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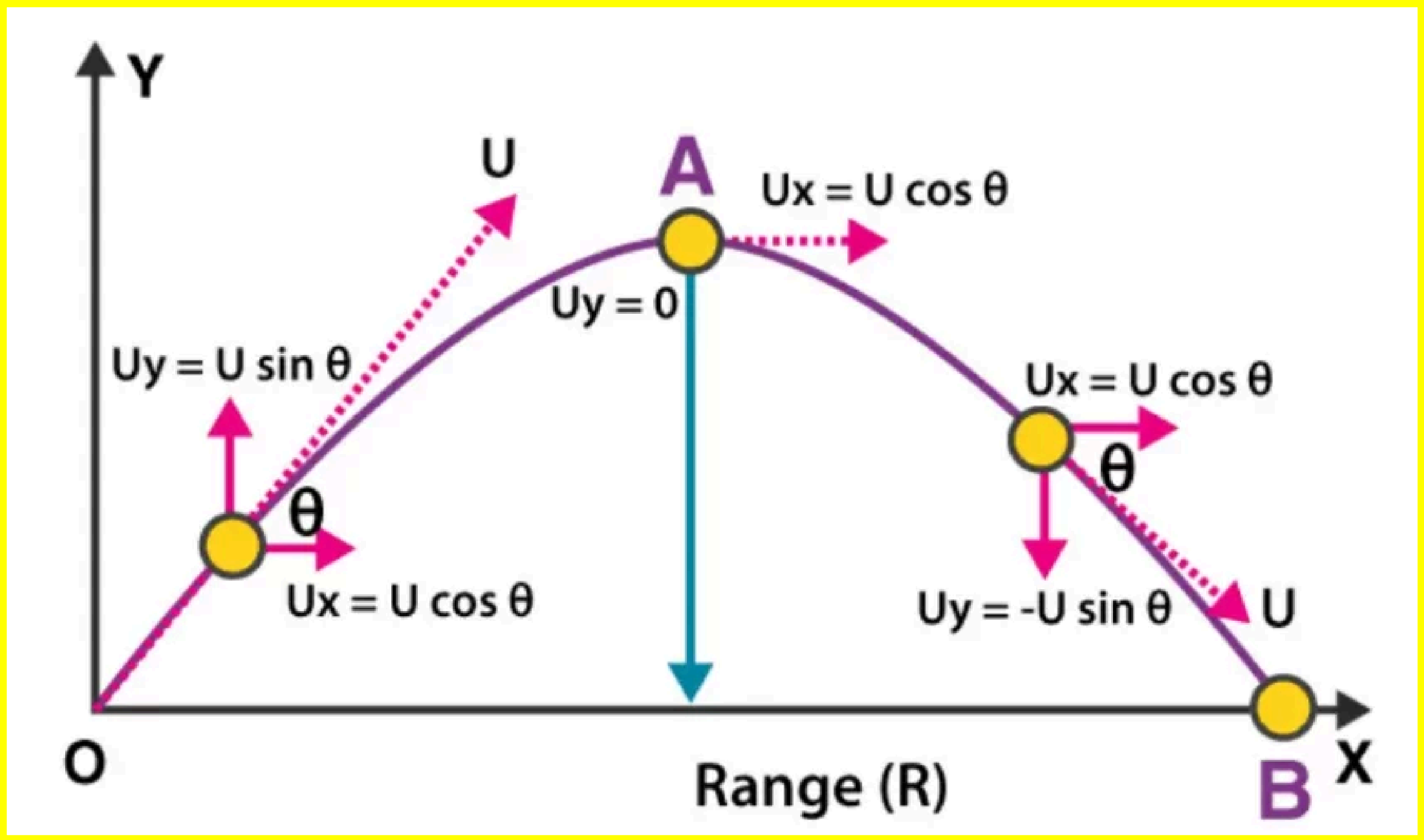
Velocity at Any Time

- Horizontal Direction (x-axis): V_x = ucosθ (remains constant)
- Vertical Direction (y-axis): V_y = usinθ – gt

Resultant Velocity:

- Magnitude: $|V| = \sqrt{V_x^2 + V_y^2}$
- Direction: $\tan\alpha = \frac{V_y}{V_x}$

$$\Rightarrow \alpha = \tan^{-1}\left(\frac{V_y}{V_x}\right)$$



Velocity Just Before Hitting the Ground

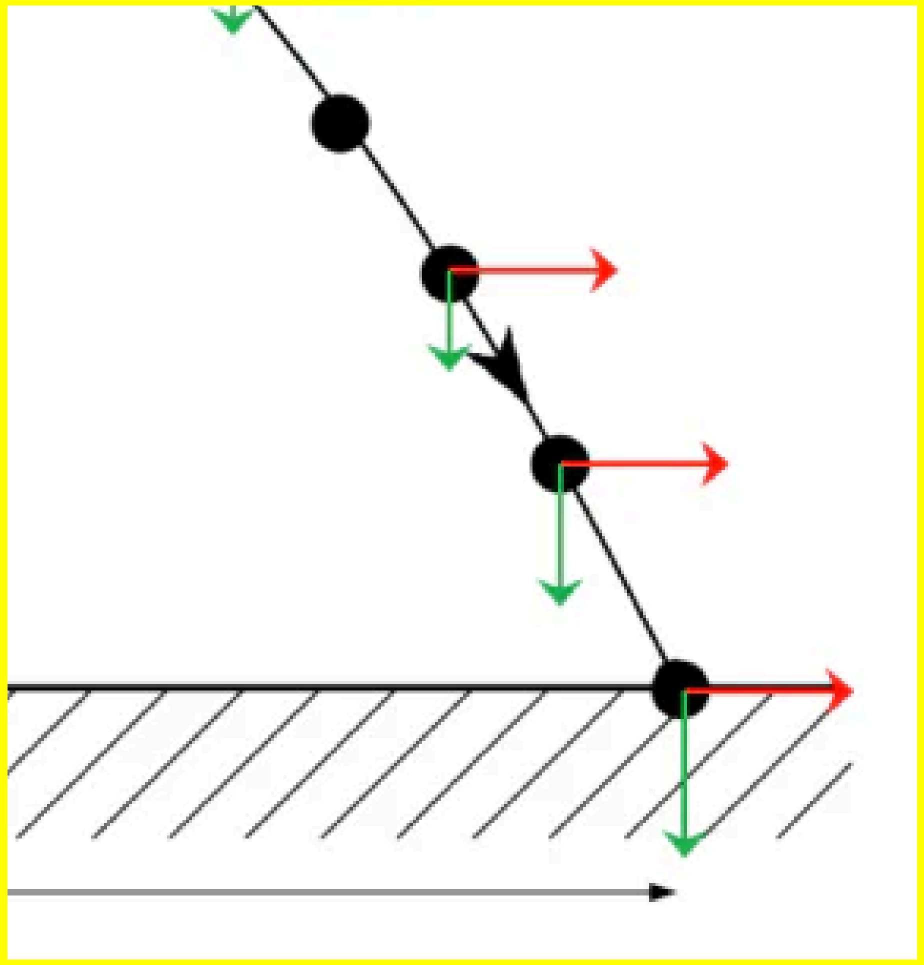
- Speed before hitting the ground = initial speed

But the direction is reversed in the vertical component.

Horizontal Component: $V_x = u\cos\theta$ (unchanged)

Vertical Component: At time t = T: $V_y = u\sin\theta - gT$

$$V_y = u\sin\theta - g\cdot\frac{2u\sin\theta}{g} = -u\sin\theta$$



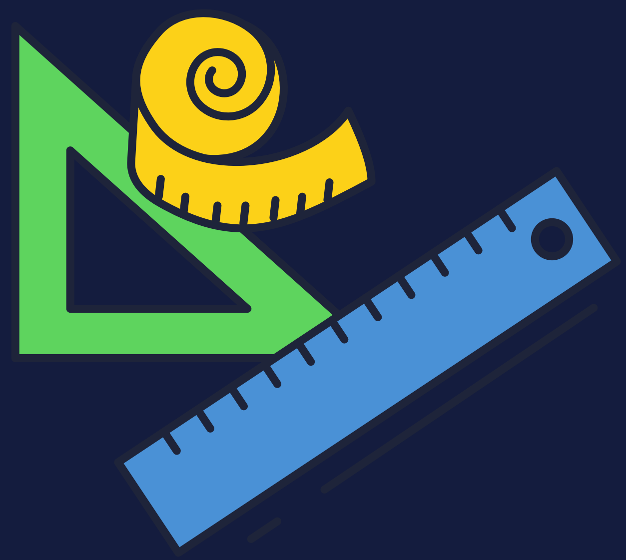
Q . A projectile reaches a maximum height H and has a range R. The ratio of time taken to reach maximum height to the total time of flight is:

A. ¼

B. ½

C. ¾

D. 1

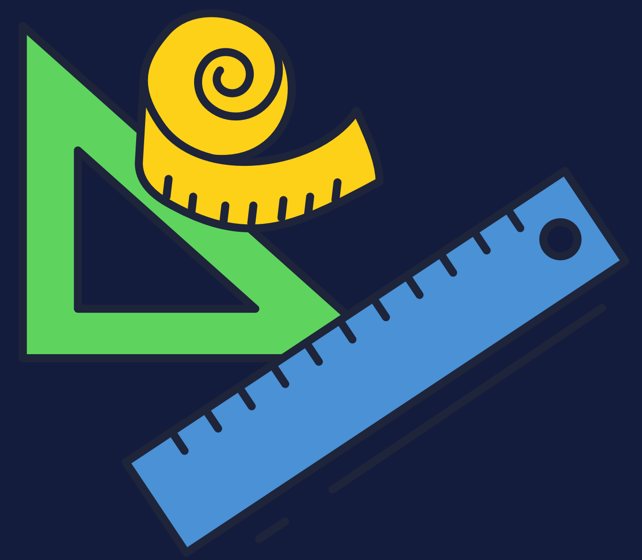


Q . A projectile is fired with a velocity of 20 m/s at an angle of 30° to the horizontal.

(a) Calculate the time of flight.

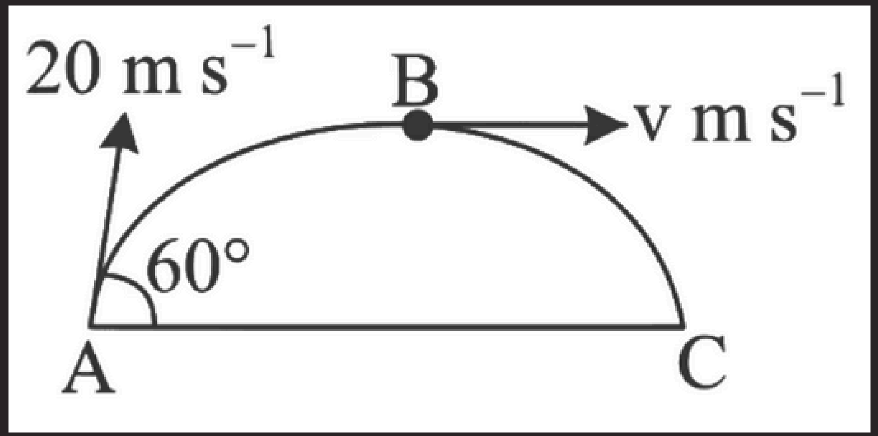
(b) Calculate the horizontal range.

(c) Calculate the maximum height reached.



Q. A ball is projected from point A with a velocity of 20 m/s at an angle of 60° to the horizontal direction.

At the highest point B of the path (as shown in the figure), the velocity v of the ball will be: [NEET 2023]



(a) 20 m/s

(b) $10\sqrt{3}$ m/s

(c) Zero m/s

(d) 10 m/s

Q . A ball is projected at a certain angle with an initial velocity u. It covers a horizontal range R. With what initial velocity must it be projected, keeping the angle of projection the same, so its horizontal range becomes 2.25R?

A. 2.5 u

B. 1.5u

C. 2.25 u

D. 0.25u

Q. An arrow is shot into the air. Its range is 200 m and its time of flight is 5 s. If g = 10 m/s², then the horizontal component of velocity of the arrow is:

(a) 25 m/s

(b) 40 m/s

(c) 31.25 m/s

(d) 12.5 m/s

Q. A stone is projected at an angle such that its horizontal range is equal to 4 times its maximum height. Find the angle of projection.

Equation of Trajectory

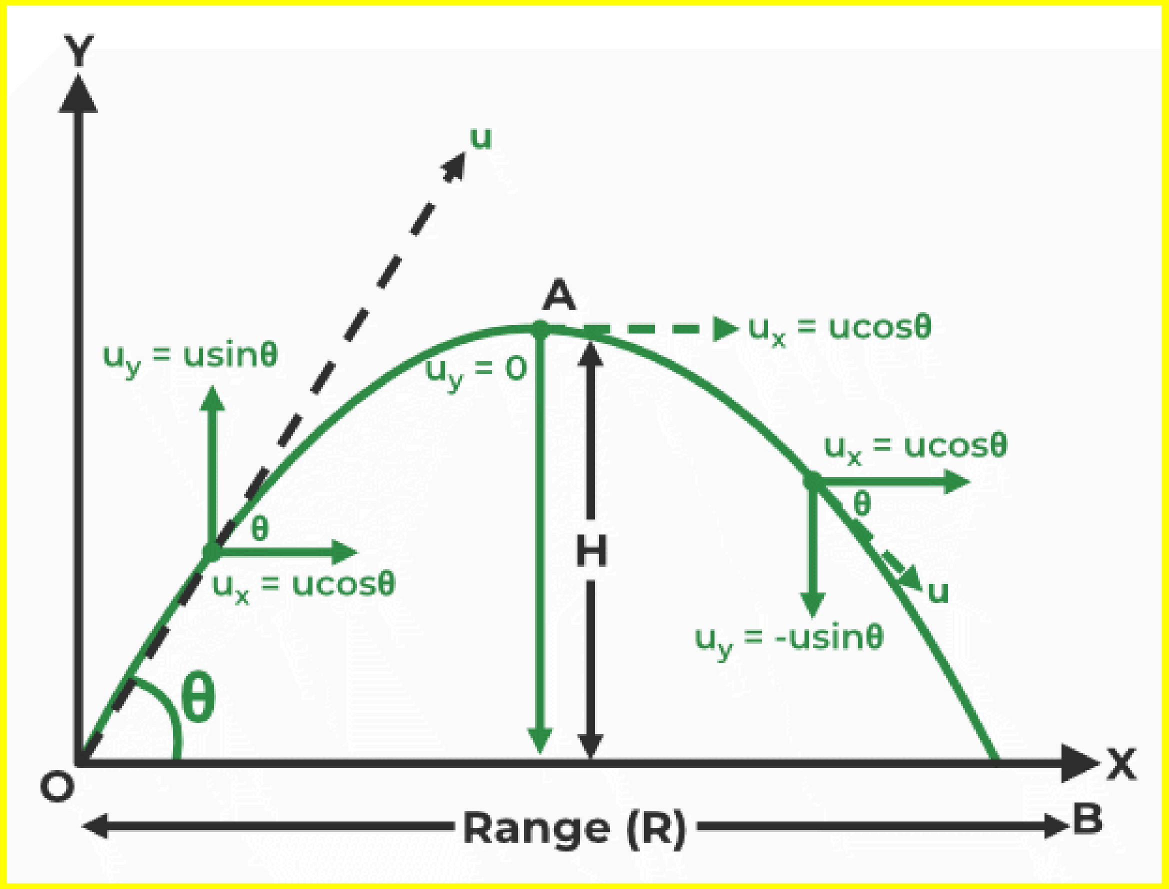
Given: Initial velocity: u

Angle of projection: θ

At time t, the projectile is at point (x,y)

Motion in the x-direction: $x = u\cos\theta \cdot t$ ....(1)

Motion in y-direction: $y = u\sin\theta \cdot t - \frac{1}{2} g t^2$ ....(2)



Deriving the Trajectory Equation:

Substitute t from (1) into (2):

From (1): $t = \dfrac{x}{u\cos\theta}$

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Substituting in (2): $y = u\sin\theta \cdot \dfrac{x}{u\cos\theta} - \dfrac{1}{2}g\left(\dfrac{x}{u\cos\theta}\right)^2$

After Simplifying, We Get: $$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

Eqn of Trajectory is in general parabolic form: $y = bx - ax^2$

Alternative Form of Equation of Trajectory (Range-Based Equation):

Equation of Trajectory: $$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

Divide both sides of the equation of Trajectory by xtanθ:

$$\frac{y}{x\tan\theta} = \frac{x\tan\theta}{x\tan\theta} - \frac{gx^2}{2u^2\cos^2\theta \cdot x\tan\theta}$$

$$\frac{y}{x\tan\theta} = 1 - \frac{gx^2}{2u^2\cos^2\theta \cdot x\tan\theta}$$

$$\frac{y}{x\tan\theta} = 1 - \frac{gx}{2u^2\cos^2\theta\tan\theta}$$

Trigonometric Identity tanθ = sinθ/cosθ ⇒ $\cos^2\theta.\tan\theta = \cos\theta\sin\theta$

Substituting, $\cos^2\theta.\tan\theta = \cos\theta\sin\theta$:

$$\frac{y}{x\tan\theta} = 1 - \frac{gx}{2u^2\cos\theta\sin\theta} \qquad ......(3)$$

Range Formula: $R = \frac{u^2\sin 2\theta}{g} = \frac{2u^2\sin\theta\cos\theta}{g}$

Reciprocal of Range Formula: $\frac{g}{2u^2\sin\theta\cos\theta} = \frac{1}{R}$

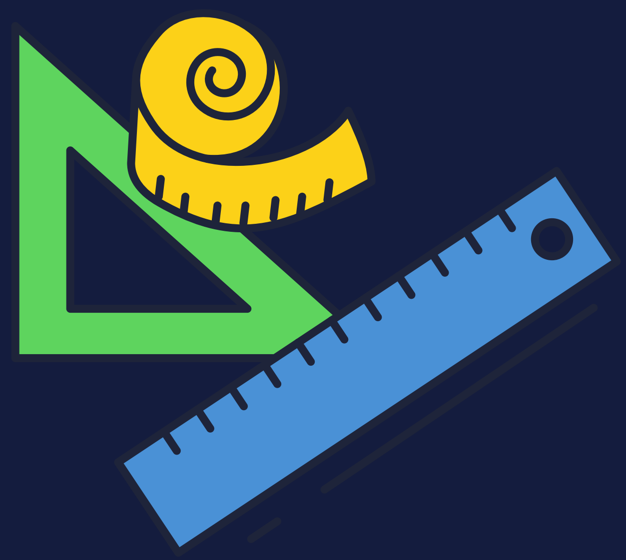
Substituting 1/R into (3): $$\frac{y}{x\tan\theta} = 1 - \frac{x}{R}$$

Final Form (After Rearranging):

$$y = x\tan\theta\left(1 - \frac{x}{R}\right)$$

This form is useful for:
Finding the range R

Q . The path of a projectile is described by the equation $y=2x-9x^2$. Find the initial speed v_0 if the projectile was launched at an angle θ_0 to the horizontal.



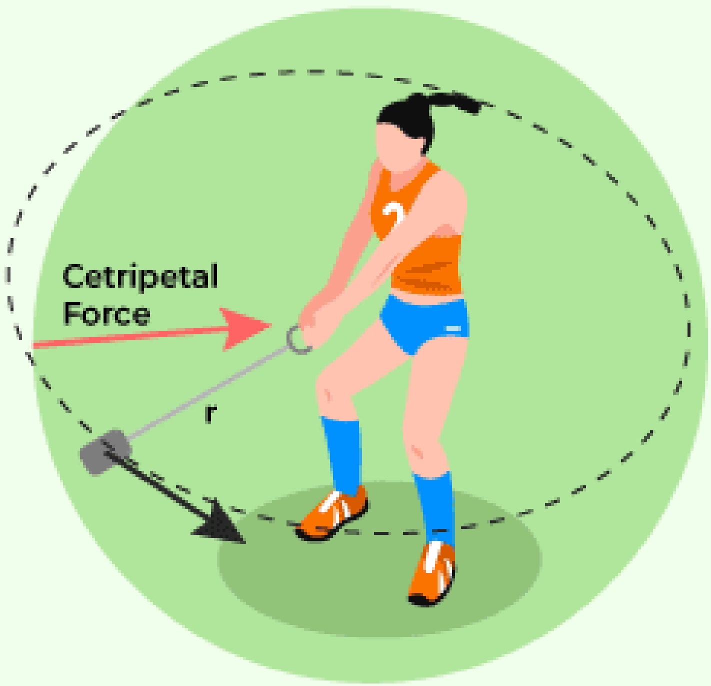
Q . The equation of a projectile is $y = \sqrt{3} \cdot x - 5 \cdot x^2$. Find the range of the projectile.

A. $\sqrt{3} / 5$

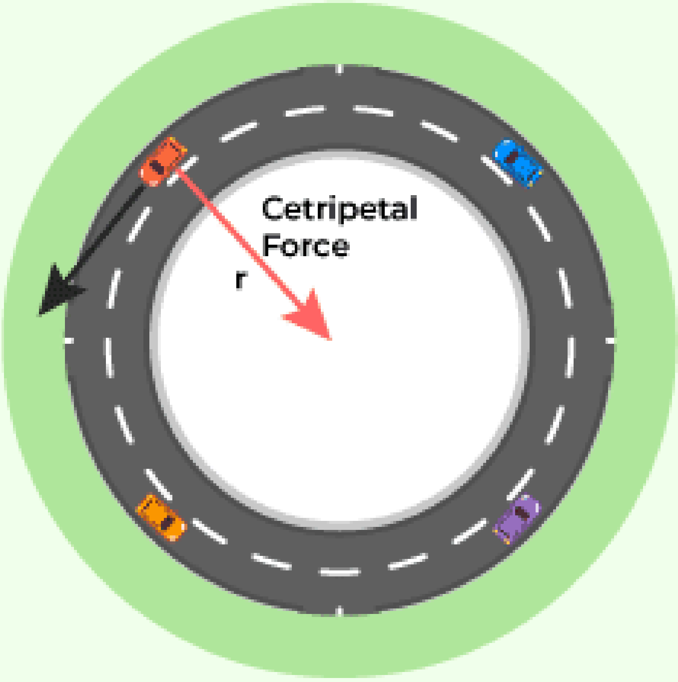
B. $5 / \sqrt{3}$

C. $5 / 3$

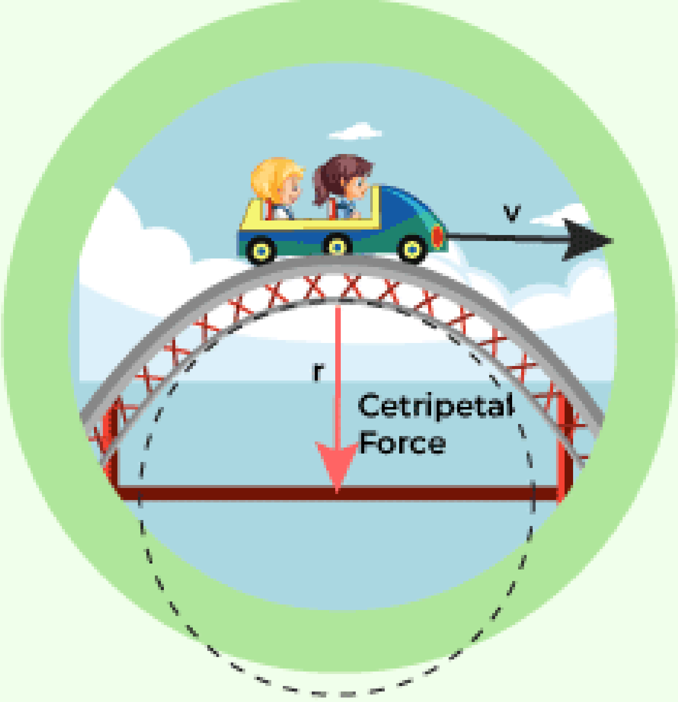
D. $3 / 5$



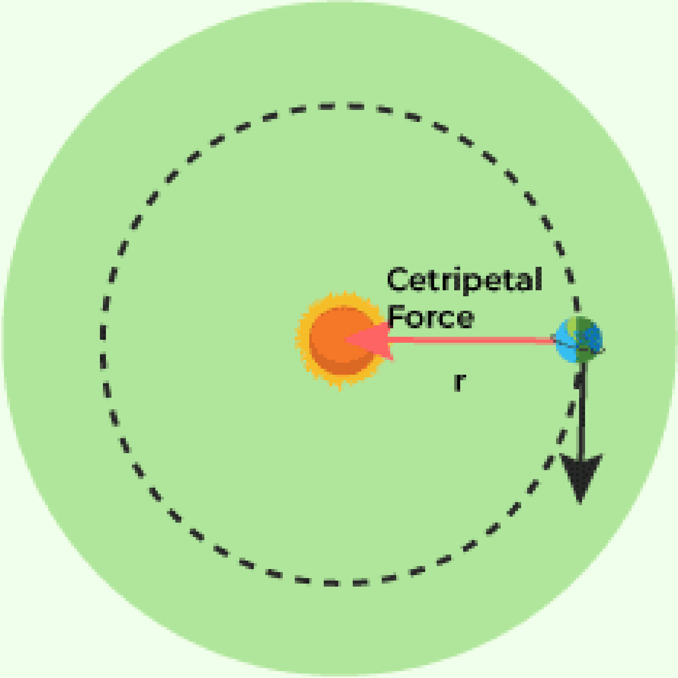
Spinning a ball on a string or twirling a lasso



Turning a car



Going through a loop on a roller coaster



Planet orbiting around the sun

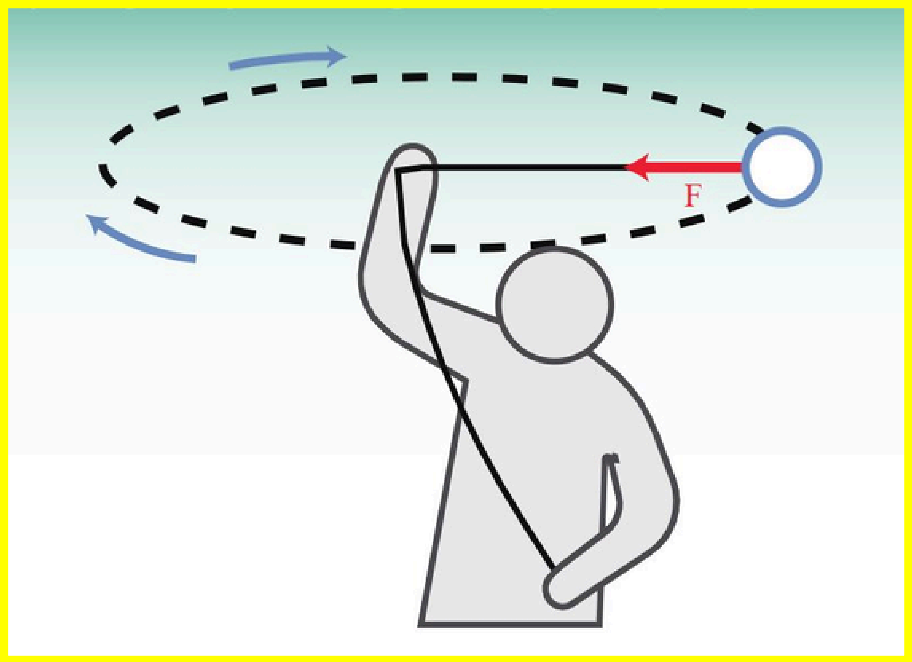
Circular Motion

Circular Motion

Circular motion refers to the motion of a particle or object along the circumference of a circle. The distance from the centre remains constant, and the direction of the object's velocity vector continuously changes.

Circular Motion is divided into:

- Circular Kinematics (Motion in a plane; does not involve causes of motion)
- Circular Dynamics (Involves causes, uses Newton's Laws of Motion (NLM))



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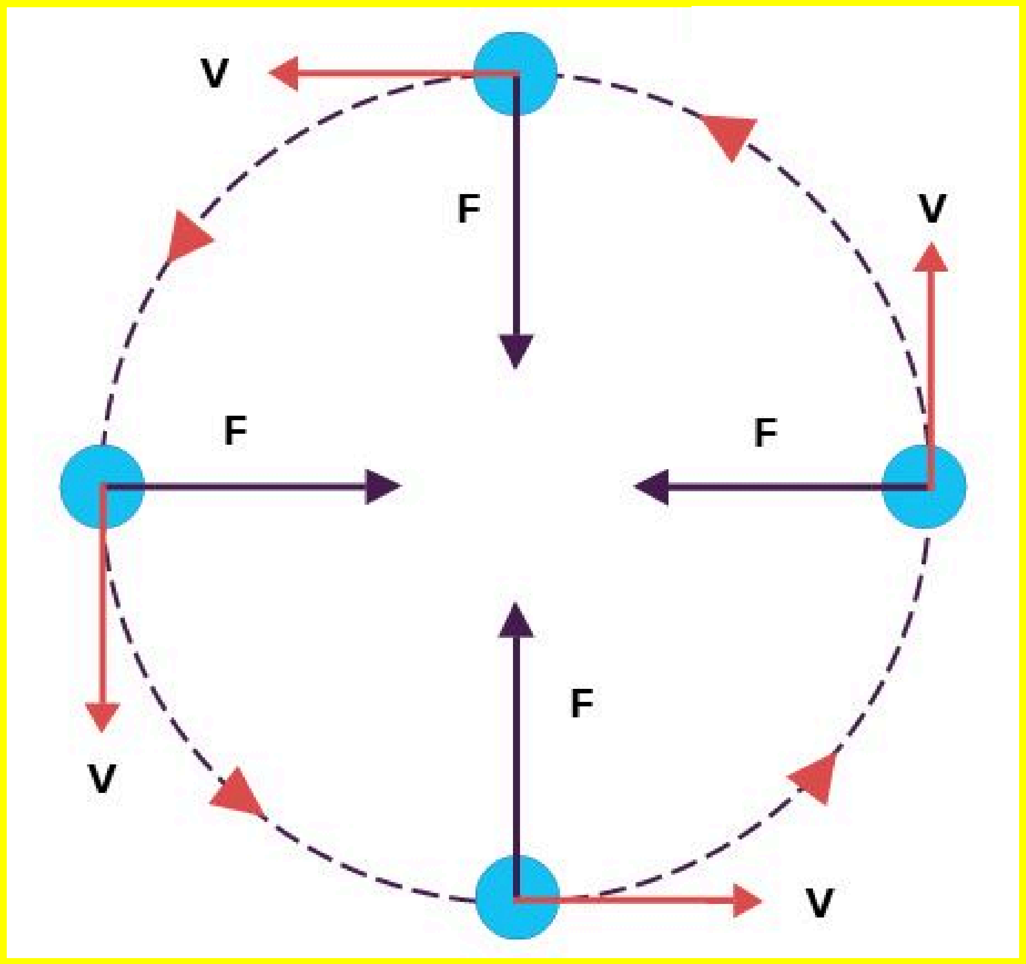
Kinematics of Circular Motion

Uniform Circular Motion (U.C.M.)

Motion with constant speed along a circular path. Direction changes, so velocity and acceleration change.

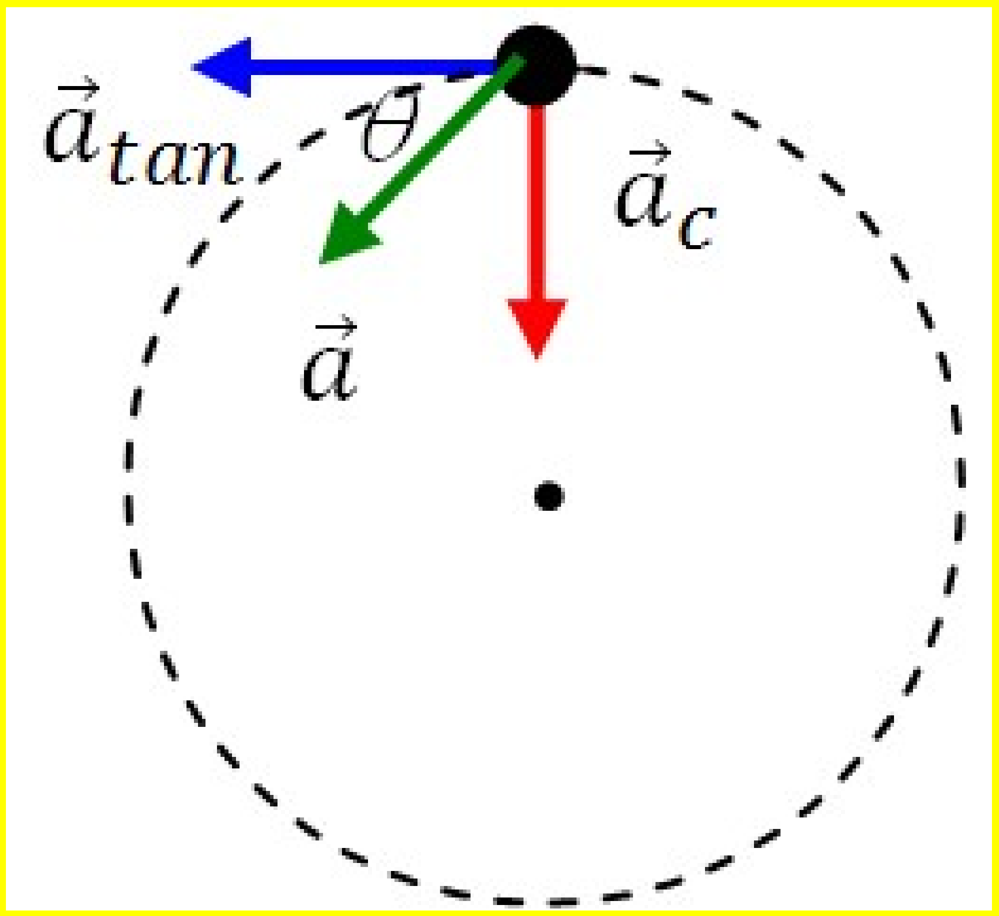
- Speed = constant
- Velocity = changes due to directional change
- Tangential Acceleration 0
- Force ≠ 0
- Direction of Acceleration & Force:
    - Centripetal (towards the center)



Non-Uniform Circular Motion (N.U.C.M.)

Motion where speed is not constant; both speed and direction of motion change continuously.

- Speed = changes
- Velocity = changes (both direction and magnitude)
- Acceleration ≠ 0
- Force ≠ 0
- Direction of Acceleration & Force has two components:
    - Tangential: due to speed change
    - Centripetal: due to direction change



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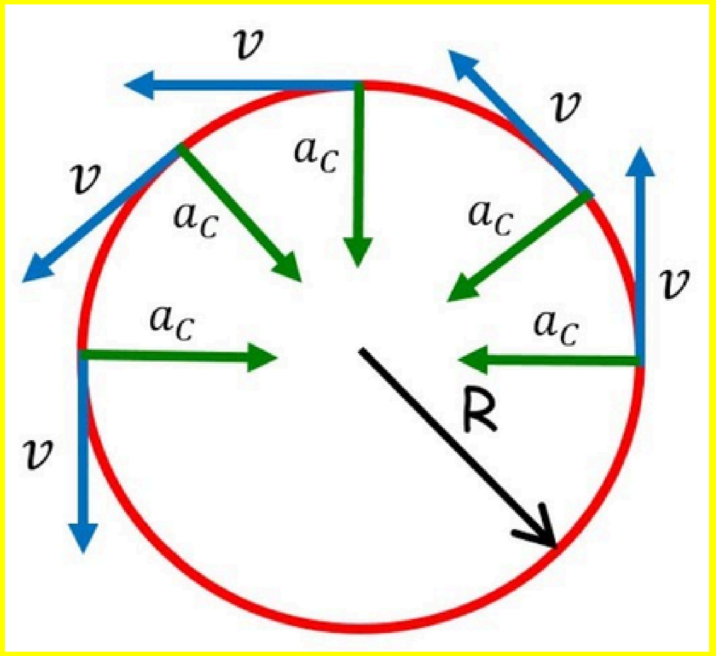
Centripetal Acceleration

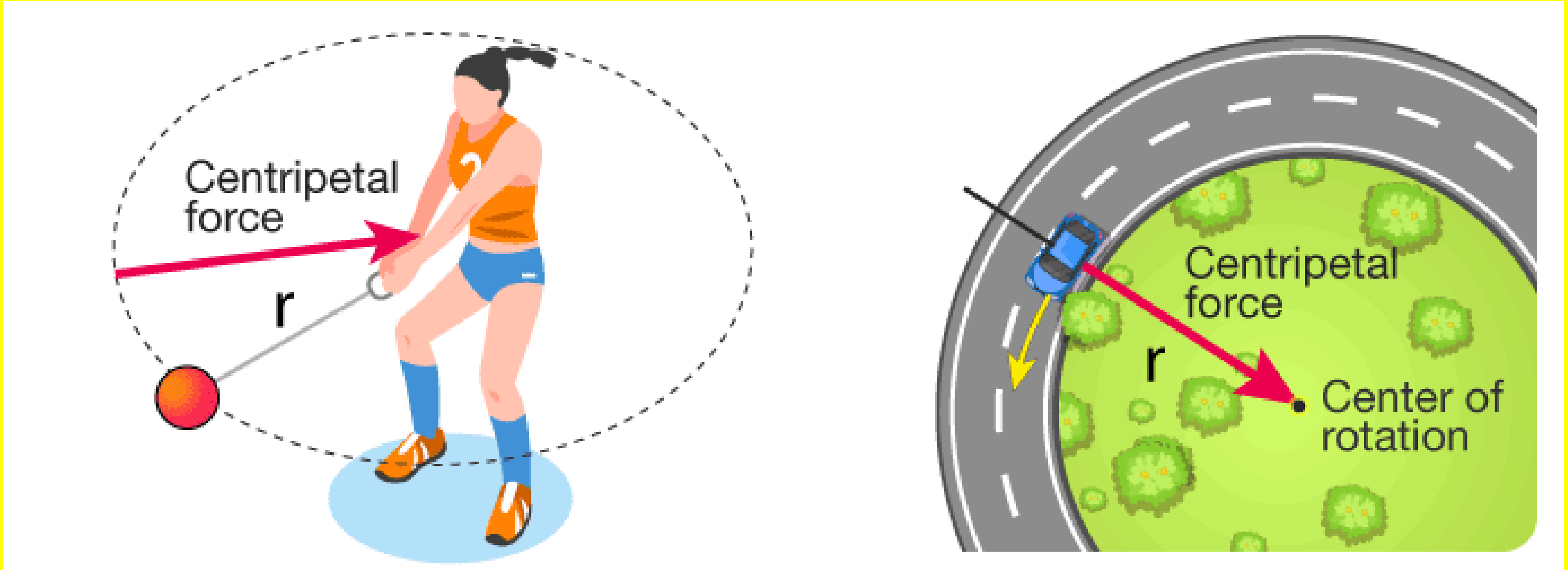
Acceleration that acts towards the centre of the circular path.

- It is also called Radial Acceleration.
- Direction is along the radius, inward.



Formula: $a_c = \dfrac{v^2}{R}$

or $a_c = \dfrac{(\text{Speed})^2}{\text{Radius}}$



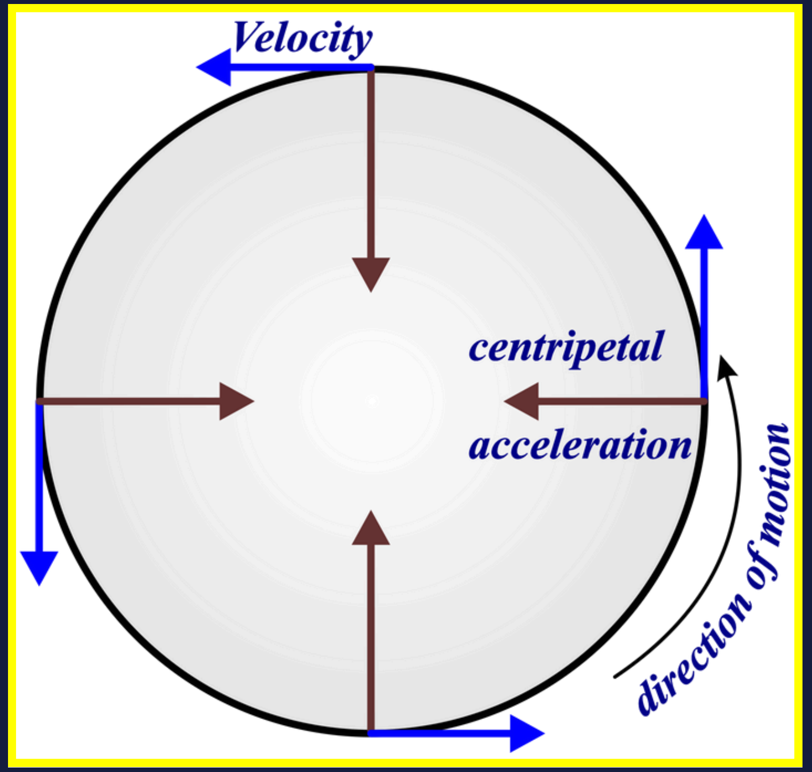
(a) Spinning a ball on a string or twirling a lasso

(b) Turning a car

Properties of Centripetal Acceleration

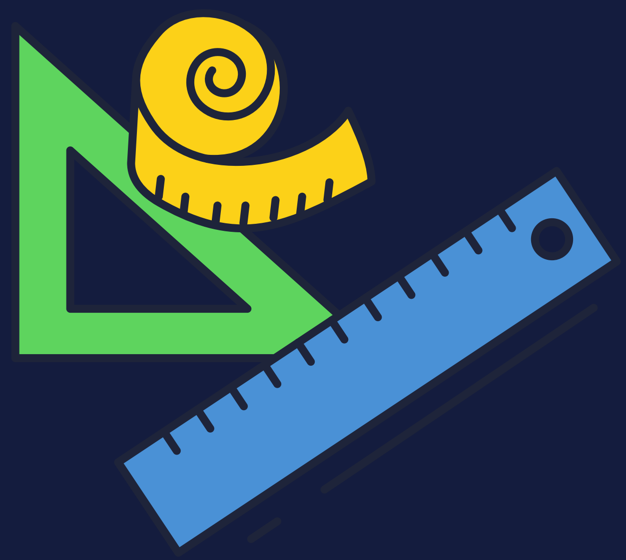
- Always acts towards the centre of the circular path.
- Always perpendicular to the velocity (i.e., to the direction of motion).
- It only changes the direction of velocity,
  - Cannot change its magnitude (i.e., does not affect speed).

Centripetal Force: $F_c = ma_c$



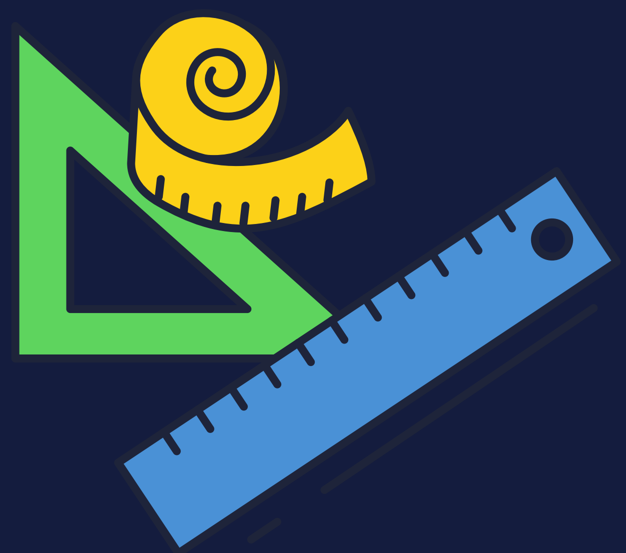
Q. If the speed of a particle in circular motion is doubled, the centripetal acceleration becomes:

A. Doubled
B. Tripled
C. Four times
D. Halved



Q . A 1 kg mass is tied to a string and rotated in a horizontal circle of radius 0.5 m at a speed of 2 m/s. Calculate:

(a) Centripetal acceleration



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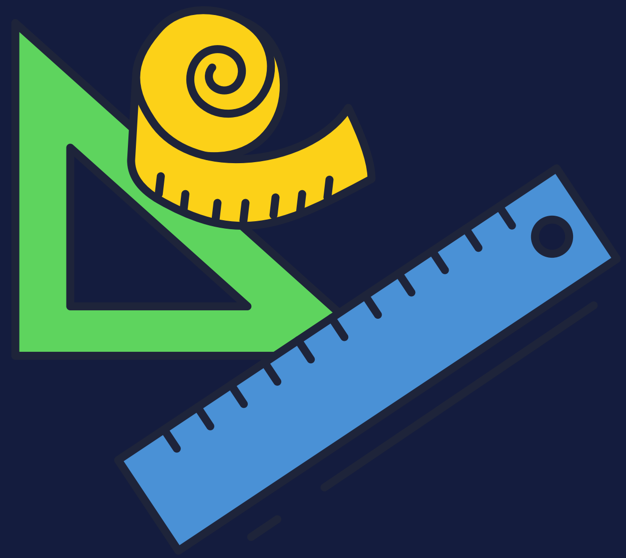
Q . A stone tied at the end of the string is whirled in a circle. If the string breaks, the stone flies away tangentially. Why?

When the string breaks, the centripetal force is removed. According to Newton's First Law, the stone moves in the direction of its instantaneous velocity, which is tangential to the circular path. Hence, it flies away tangentially due to inertia.



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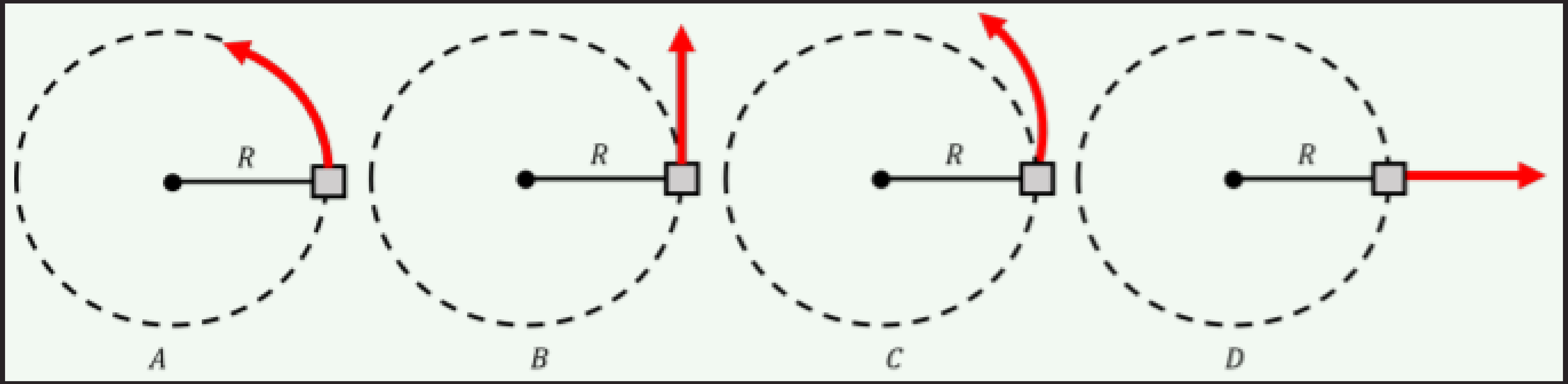
Q . An object is undergoing uniform circular motion in the horizontal plane, when the string connecting the object to the center of rotation suddenly breaks. What path will the block take after the string broke?

1. A
2. B
3. C
4. D



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Angular and Linear Variables

Position & Displacement

Angular Position and Displacement

Angular position (θ) is measured in radians (rad).

Angular Displacement ($\Delta\theta$): $\Delta\theta = \theta_2 - \theta_1$

Relation Between Linear and Angular Displacement

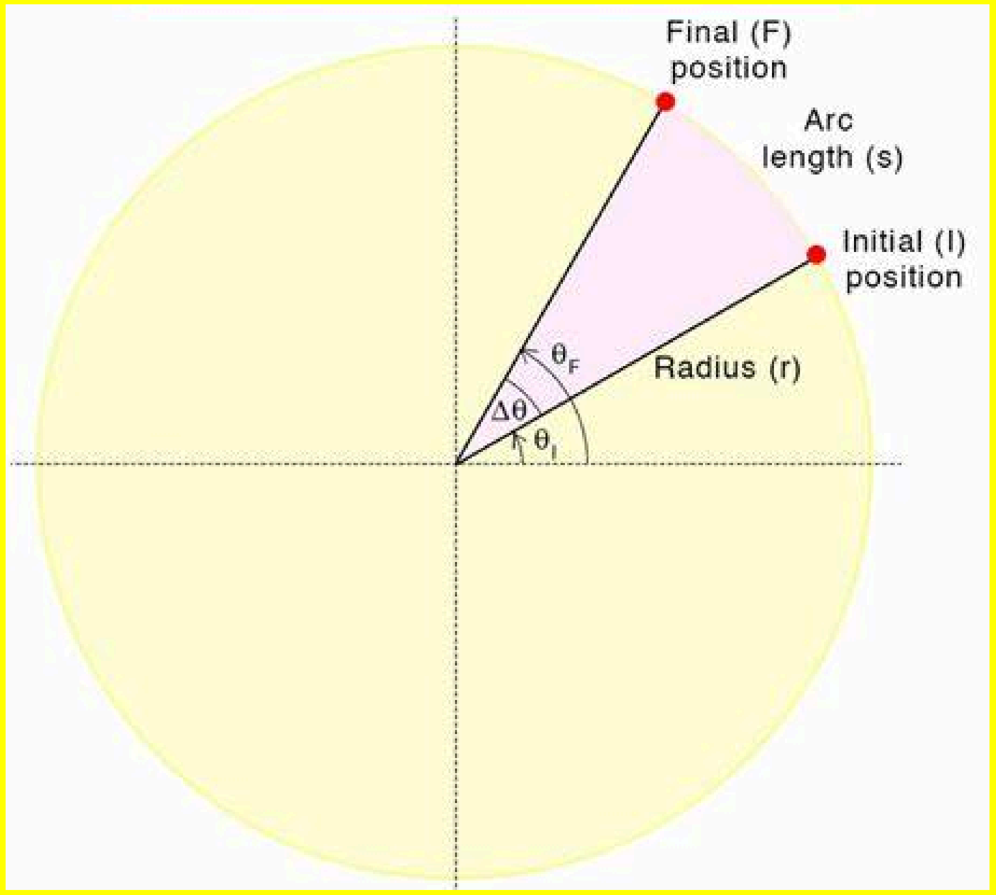
Arc Length: $L = R\theta$

Linear displacement (vector): $x = R(\Delta\theta)$

Where:

θ_1 = initial angular position

θ_2 = final angular position

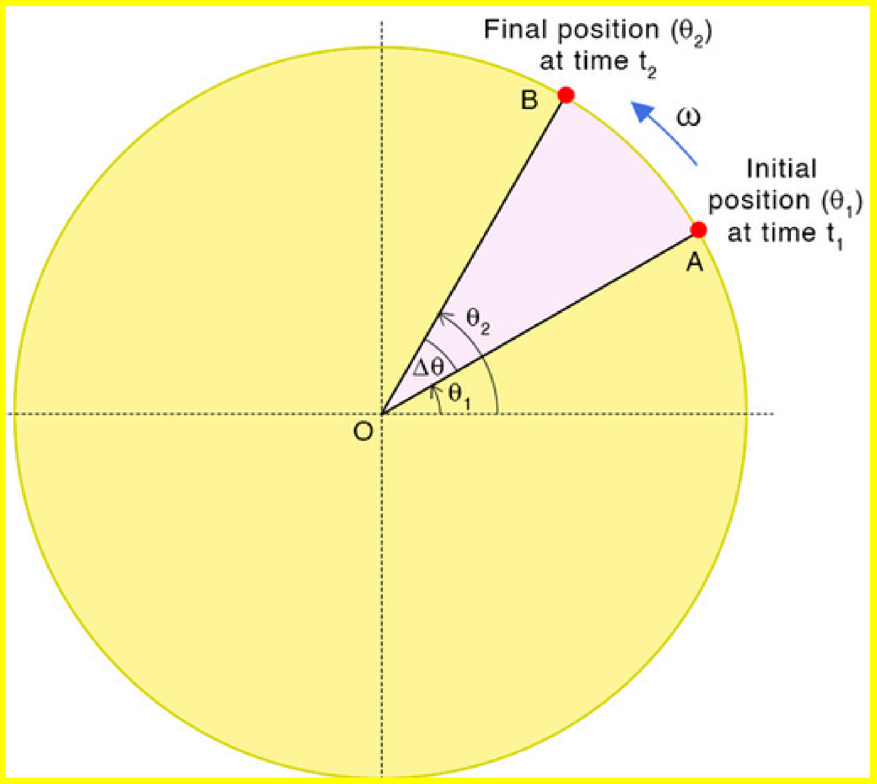


Angular and Linear Velocity

Angular velocity is the rate of change of angular displacement:

Angular Velocity (ω): $\omega = \frac{\Delta\theta}{\Delta t}$

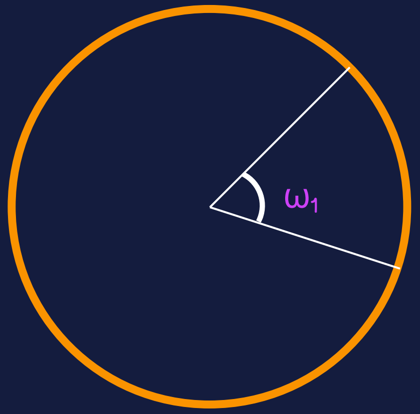
- SI unit: rad/s
- Dimension: $[T^{-1}]$

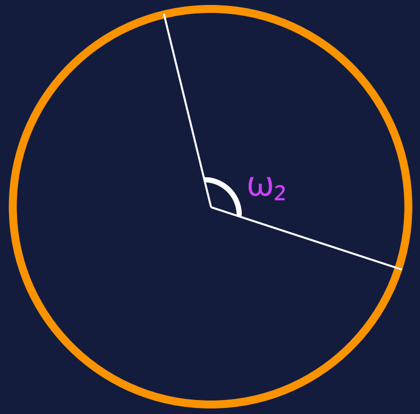


Relation Between Linear and Angular Velocity

Linear velocity (v): $v = R\omega$

- **If $\omega_2 > \omega_1$, then:**
  - **$v_2 > v_1$ (linear speeds differ if angular speeds differ)**



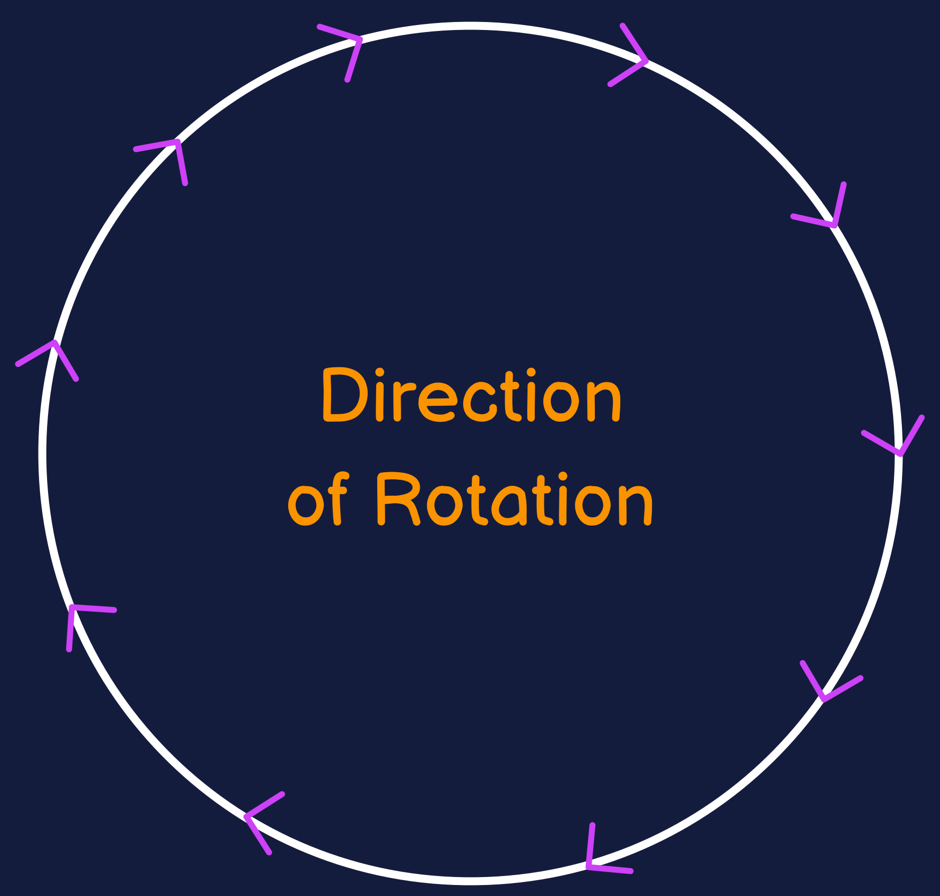


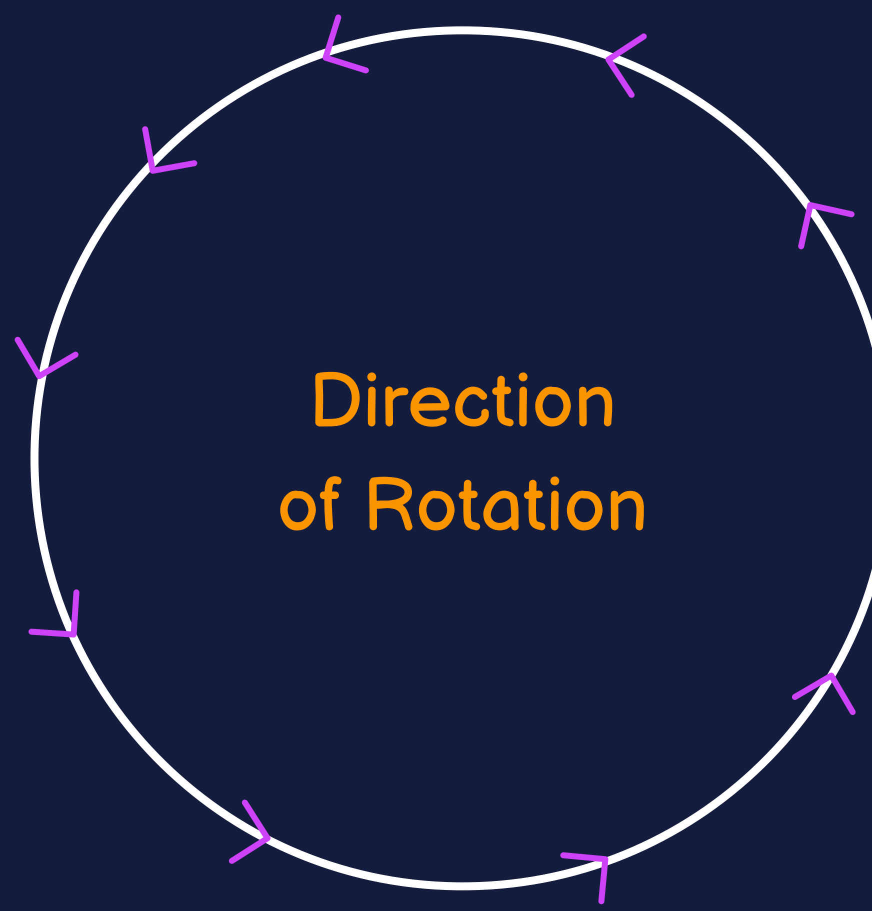
Direction of Angular Variables

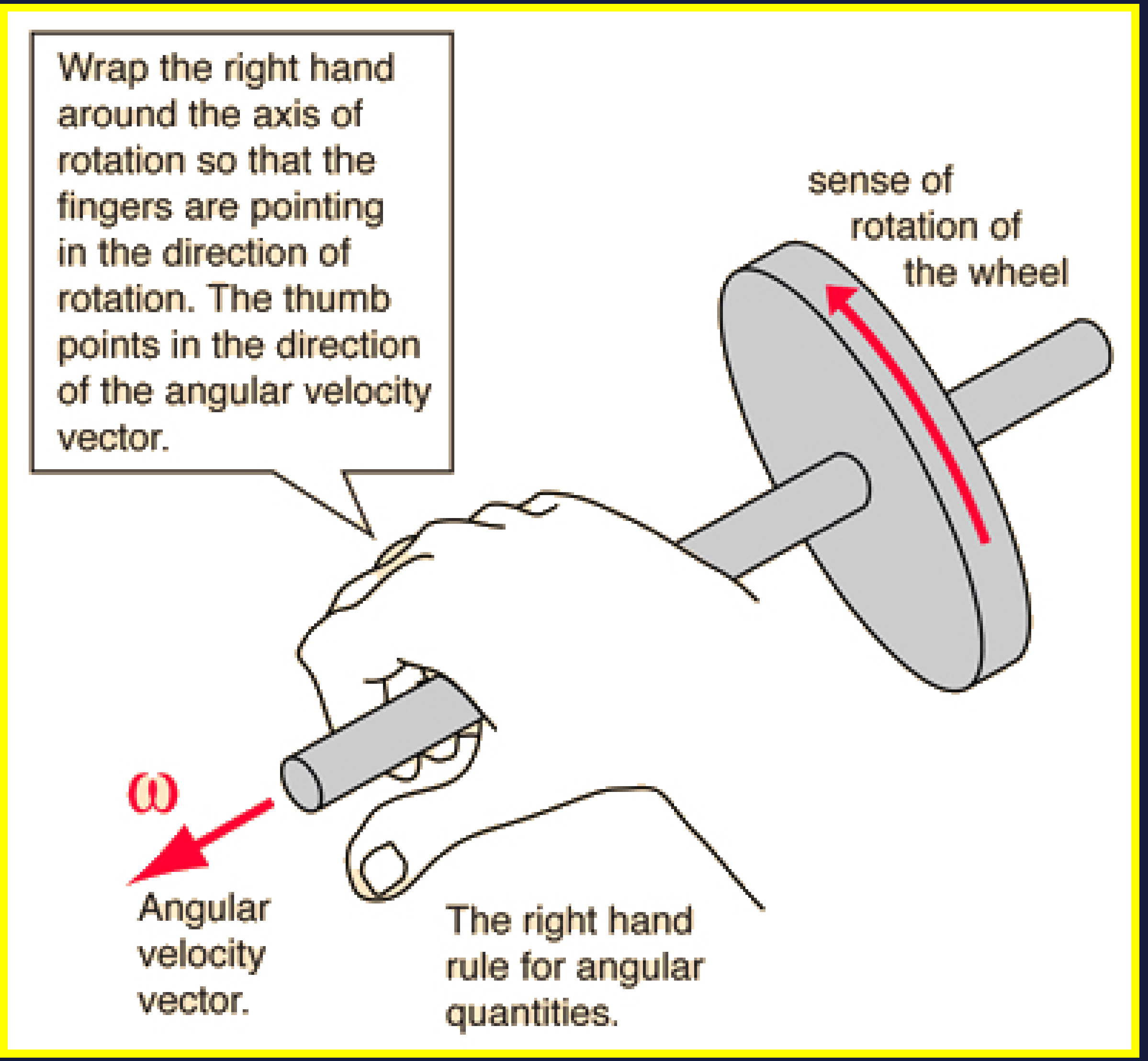
Angular displacement and velocity are vector quantities (for small angles).

Right-Hand Thumb Rule:

- Curl fingers in the direction of rotation.
- Thumb points toward the angular velocity (ω) and angular displacement (Δθ).





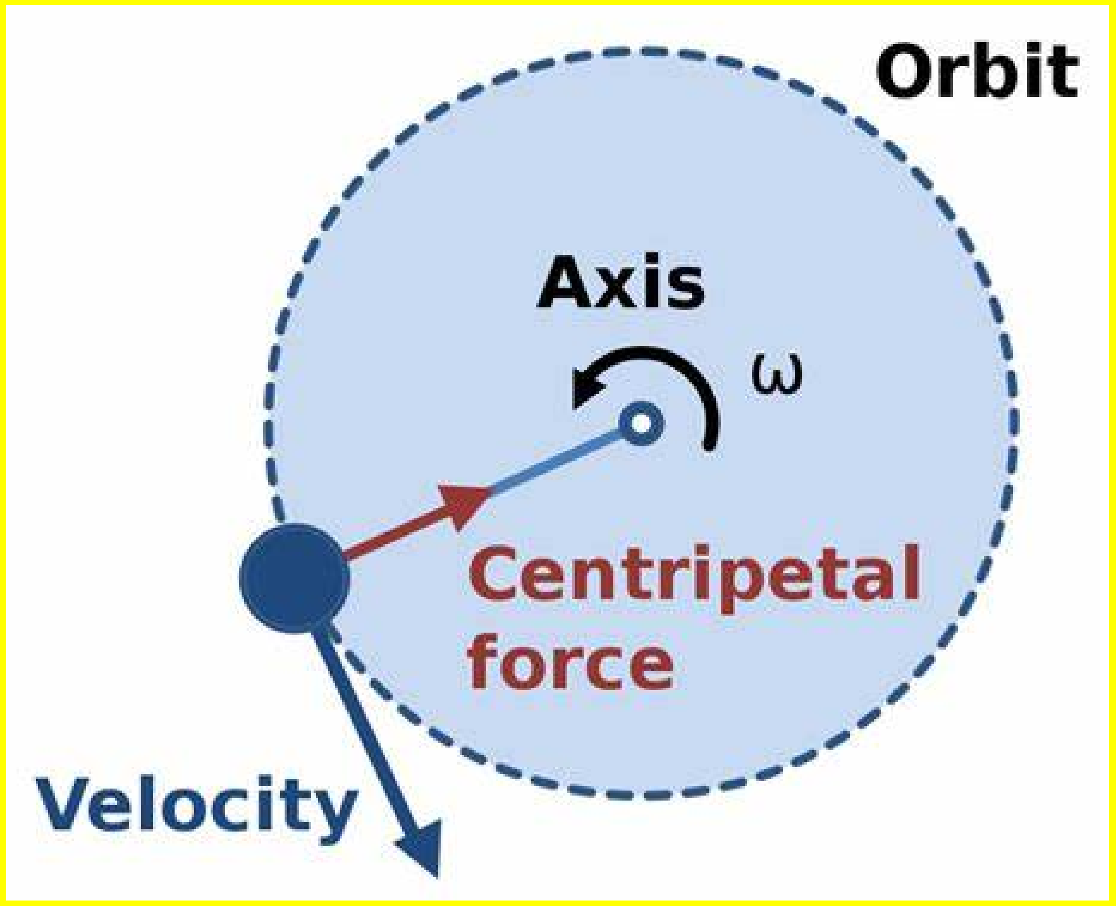


Centripetal acceleration in terms of Angular Speed

Centripetal Acceleration Formula: $a_c = \frac{v^2}{R}$

Angular Velocity(Speed): v = Rω

Substituting: $a_c = \frac{(R\omega)^2}{R} = R\omega^2$



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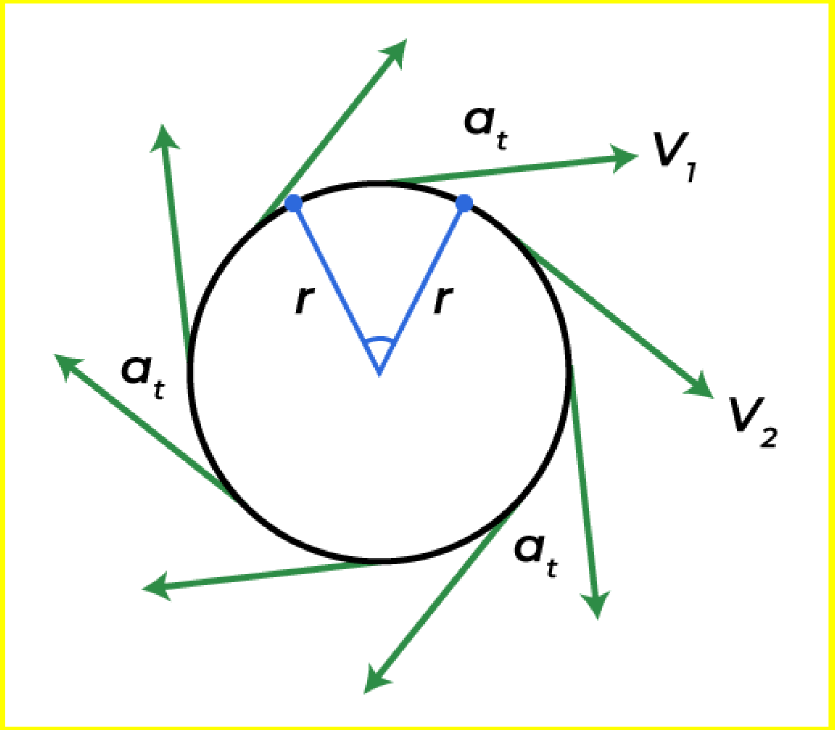
- In Uniform Circular Motion (UCM):
  - Angular speed (ω) is constant
- In Non-Uniform Circular Motion (NUCM):
  - Both angular (ω) and linear velocity (v) change



- Tangential Acceleration (a_t):
  - Additional acceleration is needed in non-uniform circular Motion.
  - Occurs due to the change in speed.

In UCM: $a_T = 0$, $a_c \perp v$, only centripetal acc.

NUCM: $a_T \neq 0$, total acceleration is diagonal → $\vec{a}_{net} = \sqrt{a_c^2 + a_T^2}$

Angular and Linear Acceleration

Angular Acceleration (α):

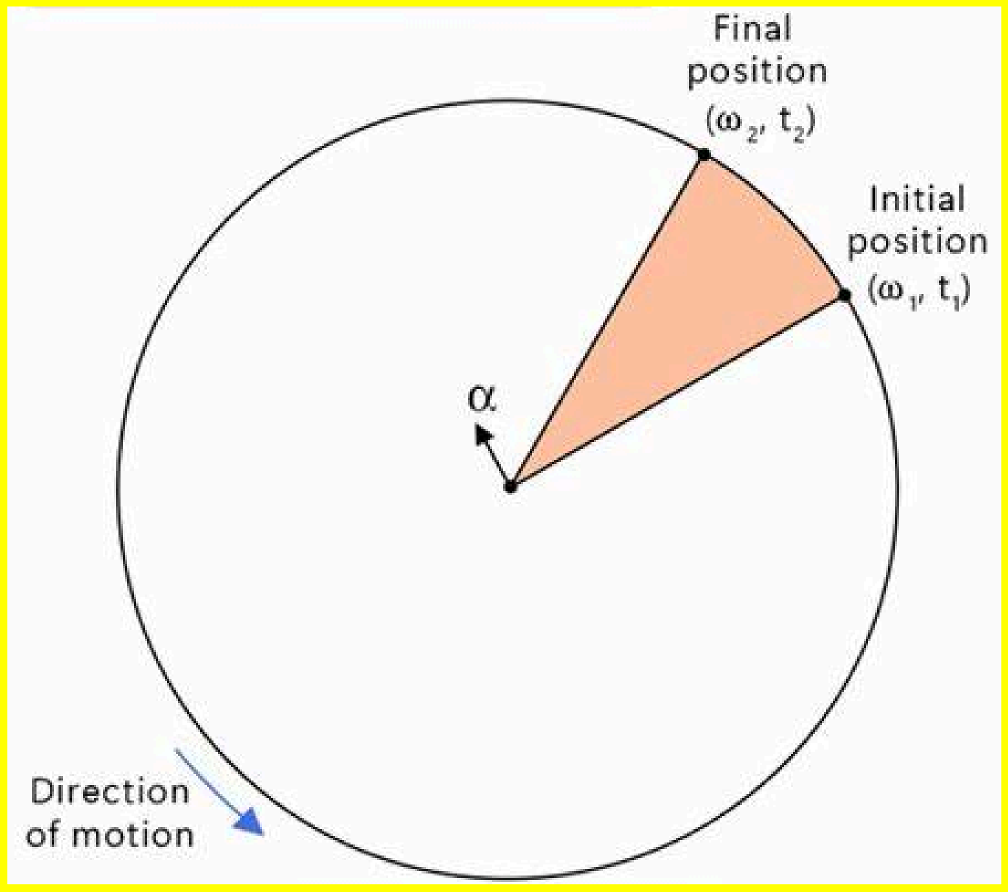
Rate of change of angular velocity

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

SI unit: rad/sec²

Linear (Tangential) Acceleration (a_t):

$$\Delta v = R\Delta\omega \Rightarrow \frac{\Delta v}{\Delta t} = R \cdot \frac{\Delta\omega}{\Delta t} \Rightarrow a_T = R\alpha$$



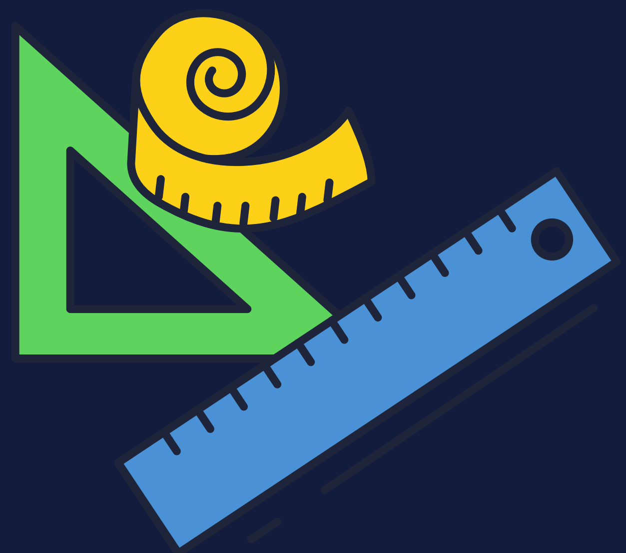
Q . A body is whirled in a horizontal circle of radius 20 cm. It has an angular velocity of 10 rad/s. What is its linear velocity at any point on the circular path?

A) 20 m/s

B) $\sqrt{2}$ m/s

C) 10 m/s

D) 2 m/s



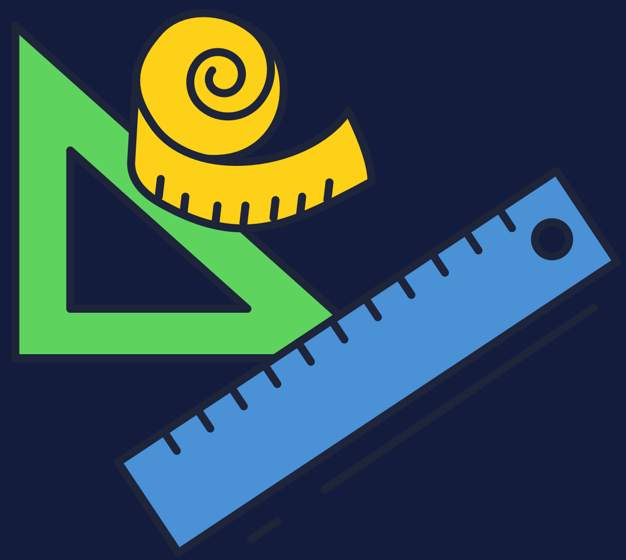
Q. If the angular velocity ω of a rotating object doubles, its centripetal acceleration will:

(A) Remain the same

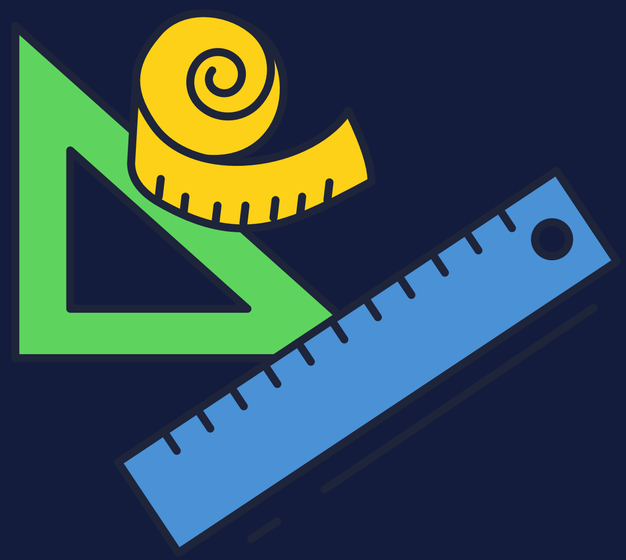
(B) Double

(C) Become four times

(D) Become half



Q. A cyclist is riding with a speed of 5 m/s. As he approaches a circular turn on a road of radius 60 m, he applies brakes and reduces his speed at a constant rate of 0.2 m/s^2. What is the magnitude and direction of the net acceleration of the cyclist on the circular turn?

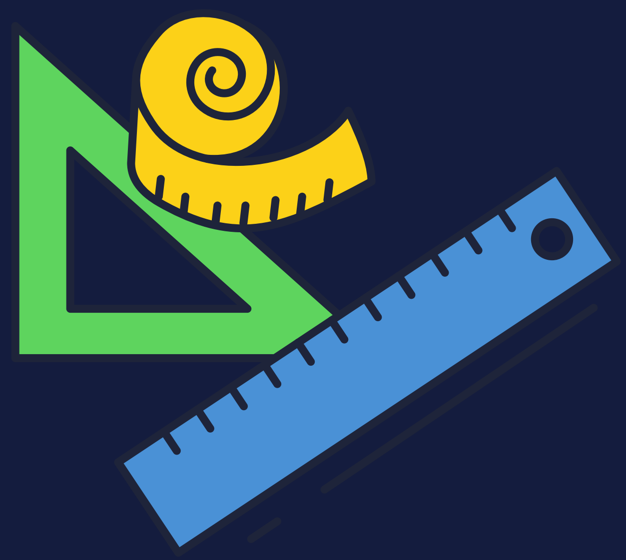


Q. An insect trapped in a circular groove of radius 12 cm moves along the groove steadily and completes 7 revolutions in 100 s.

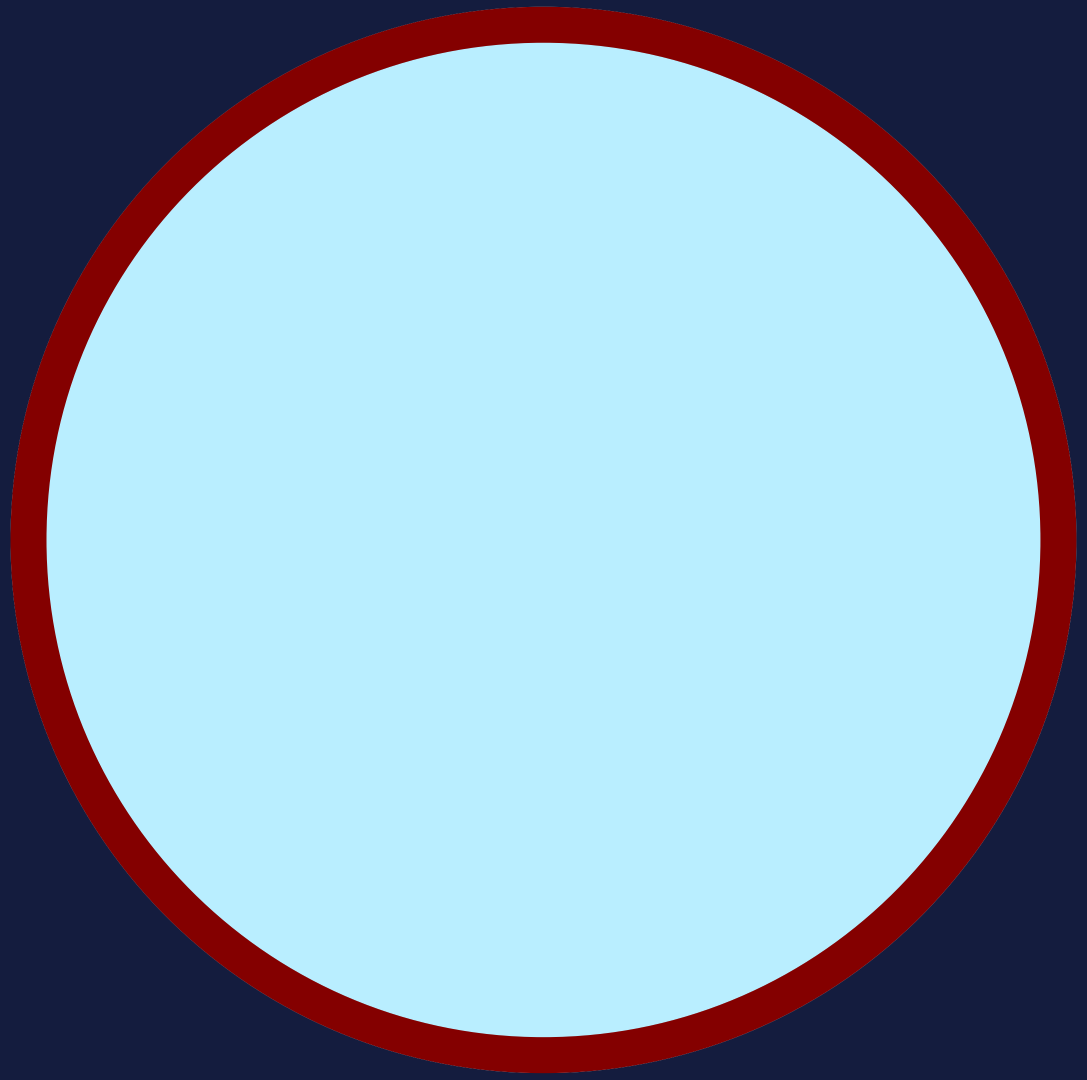
(a) What is the angular speed, and the linear speed of the motion?

(b) Is the acceleration vector a constant vector ? What is its magnitude ?

[NCERT]



Deriving Formula for Centripetal Acceleration



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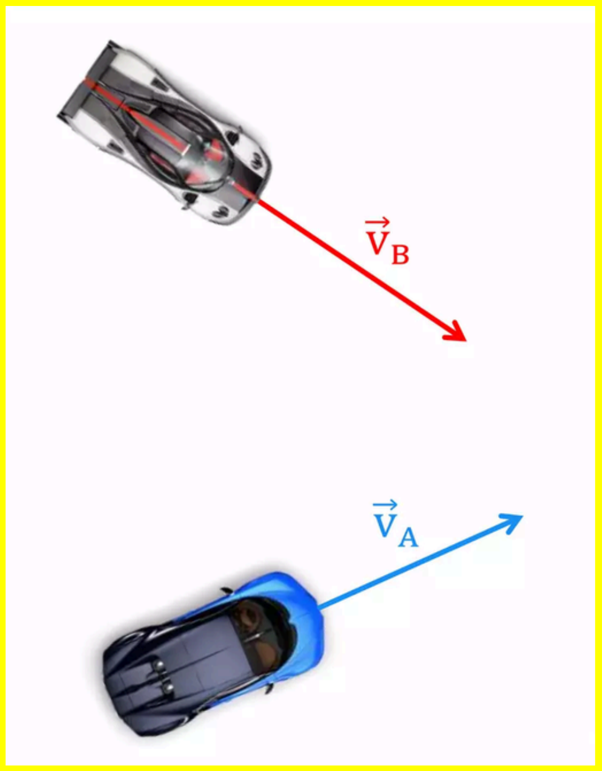
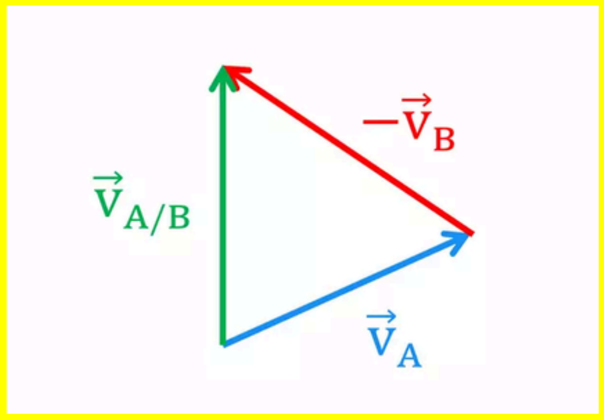
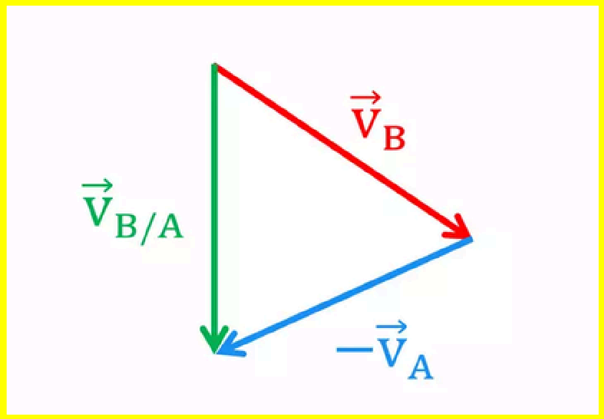
Relative Motion in Two Dimension

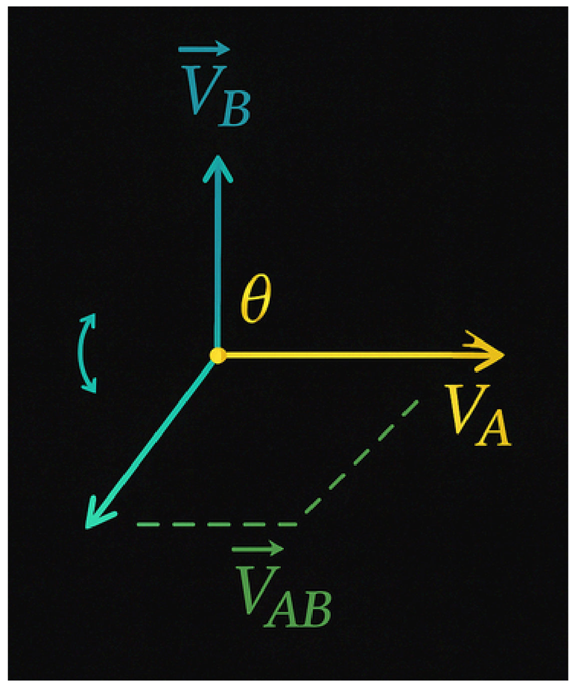


$$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$$



$$\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$$





Magnitude of Relative Velocity:

$$|\vec{V}_{AB}| = \sqrt{V_A^2 + V_B^2 - 2V_AV_B\cos\theta}$$

Graphical Representation:

- Velocity vectors form a triangle when placed tail-to-tail.
- Resultant vector = relative velocity.
- The Triangle Law / Parallelogram Law is used for vector addition.

Rain Man Problem

The "Rain Man Problem" is a classic example of relative motion. It explores how the direction and speed of rain appear to change for a person (the "man") who is moving, and how to calculate the angle or velocity at which rain hits the person.

Relative Velocity of Rain w.r.t Man:

- v_r = velocity of rain (actual)
- v_m = velocity of man

$$v_{rm} = v_r - v_m$$

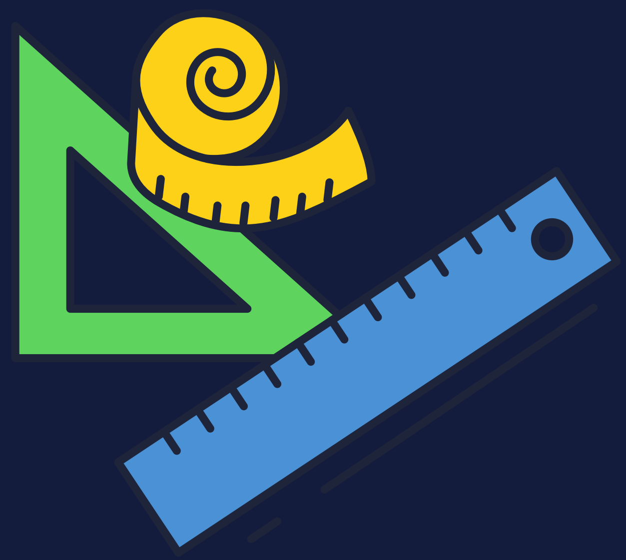
Q A man standing still observes that rain is falling vertically. When he starts running eastward at 5 m/s, he observes that the rain makes an angle of 45° with the vertical. The actual speed of rain is:

(a) $5\sqrt{2}$ m/s

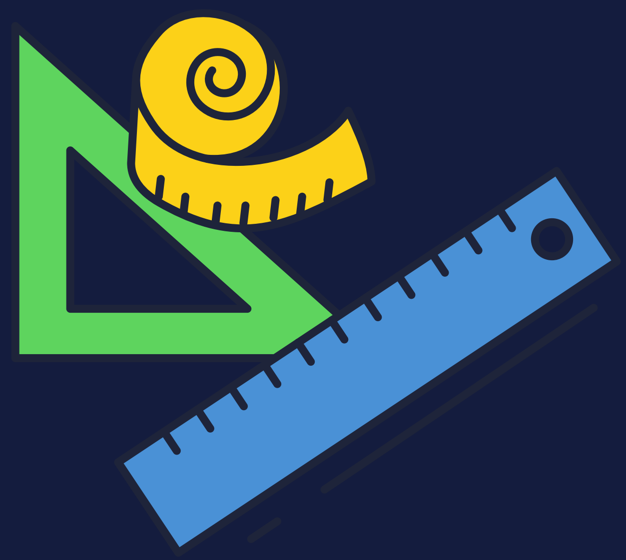
(b) 10 m/s

(c) 5 m/s

(d) 2.5 m/s



Q. Rain is falling vertically with a speed of 10 m/s. A boy runs horizontally with a speed of $10\sqrt{3}$ m/s. At what angle from the vertical should he hold his umbrella to avoid getting wet? [NEET 2018]

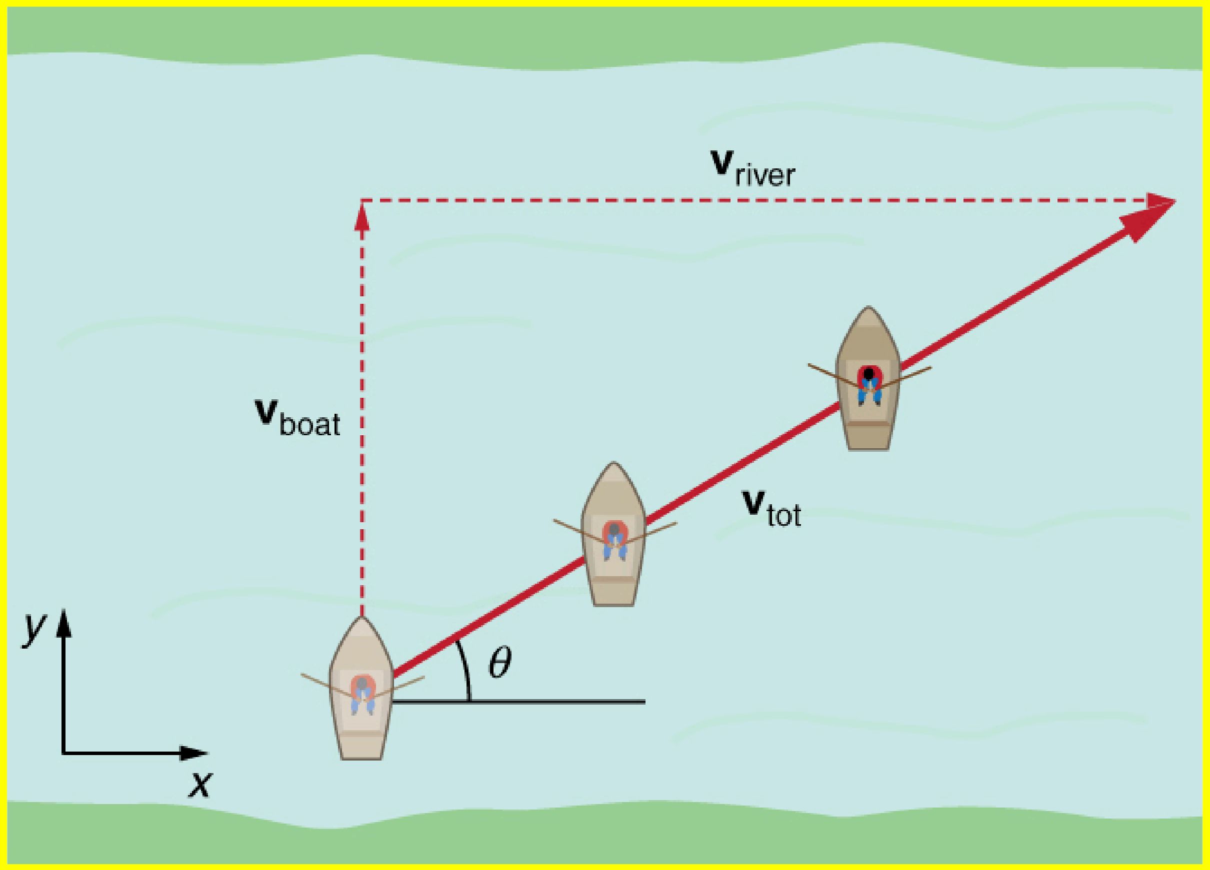


Q. Rain is falling vertically downwards with the velocity of 3km/h , a man walks in the rain with 4km/h , the rain drop will fall on the man with velocity

Q Imagine yourself in a rain steadily falling vertically with a speed of 2m/s.
If you start moving with 1m/s due east, in which direction should you hold the umbrella to protect yourself from the rain?

River Boat Problems in 2-D

- V_B: Velocity of the boat in still water (own velocity).
- V_R: Velocity of the river stream.
- V_{BR}: Velocity of the boat relative to river.



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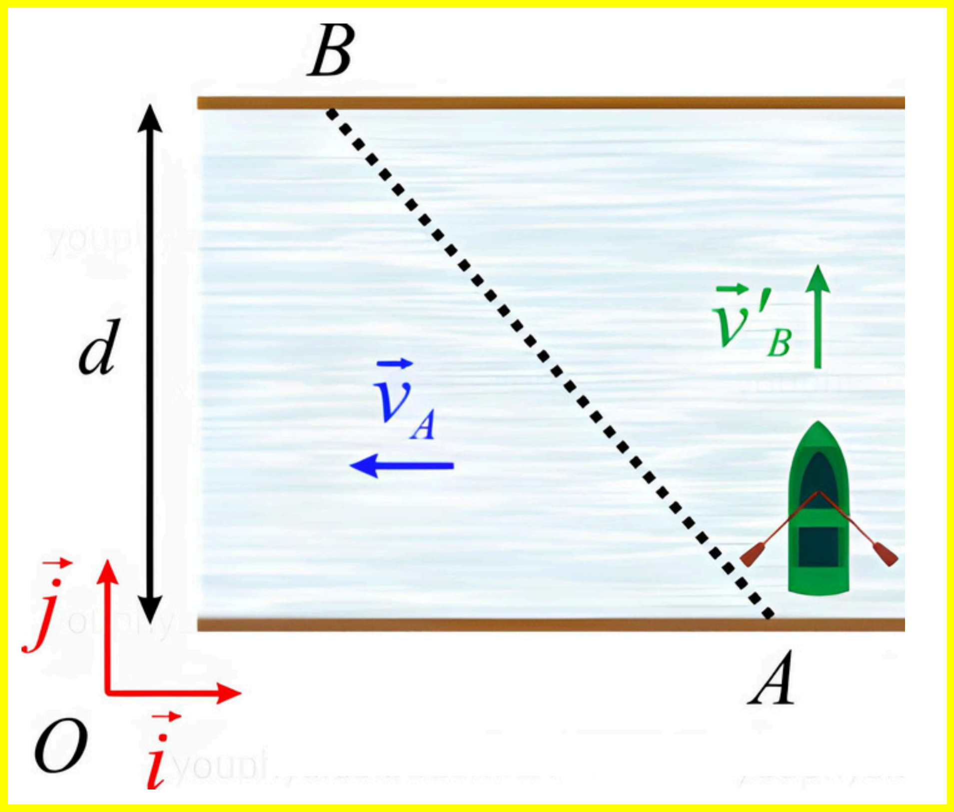
Case 1: Minimum Time to Cross

The boat heads perpendicular to the river (no horizontal effort).

Time Taken: $$t = \frac{d}{V_{BR}}$$ (where d is river width)

Drift: $$x = V_R \cdot t = V_R \cdot \frac{d}{V_{BR}}$$



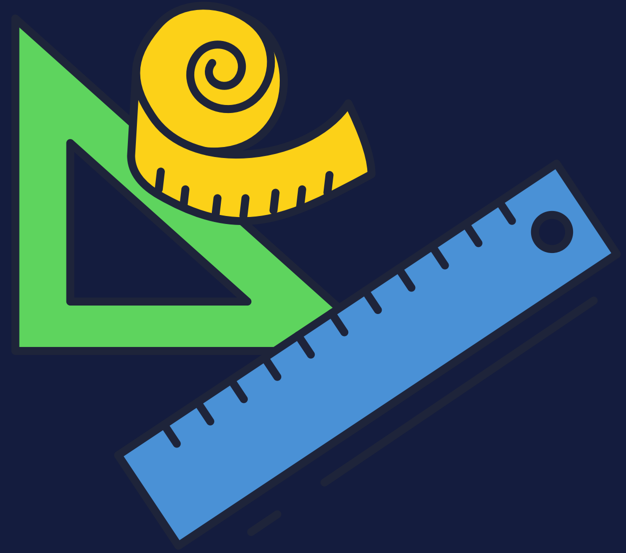
Case 2: Minimum Distance (Shortest Path)

The boat moves such that it reaches a point directly opposite (no drift).

Condition: $$V_{BR} \sin\theta = V_R$$

Time to Cross Remains: $$t = \frac{d}{V_B}$$

Q A man can swim with a speed of 4.0 km/h in still water. How long does it take to cross a river 1km/h wide if the river flows steadily at 3.0km/h and he makes his strokes normal to the river current? How far down the river does he go when he reaches the other bank?

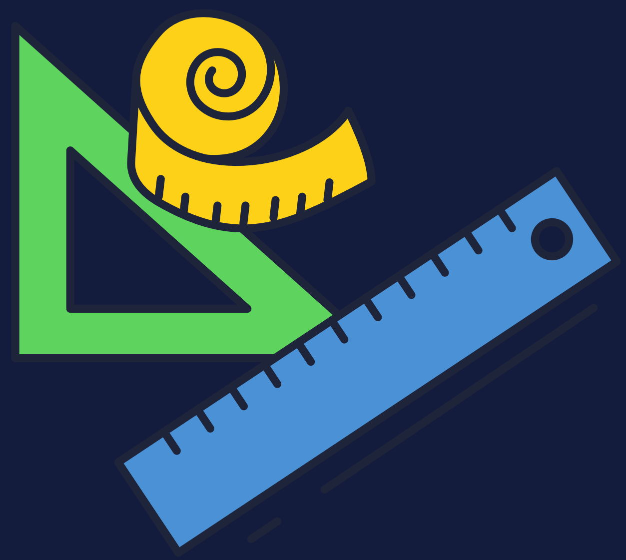


Q A boat can move with a speed of 10 km/h in still water. If the river is flowing at 5 km/h, find the time taken by the boat to cross a river 600 m wide when:

(a) It moves perpendicularly to the river flow

(b) It moves with minimum time

(c) It moves to reach a point directly opposite on the other bank



"Break study problems into components - just like vectors."

Tackle the horizontal (concepts) with logic and the vertical (stress) with patience. Then add them up to stay on the resultant path to success.